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A STATISTICAL ASSESSMENT OF THE WIND ENERGY POTENTIAL OF SOUTH-EAST NIGERIA USING THE WEIBULL MODEL

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Abstract
This paper analyses the wind energy potential of south – eastern Nigeria using monthly average wind speed observations obtained from the region for the period (1987 – 2019). The 2-parameter Weibull distribution was used to fit the observed wind speed observations and it was found to be a highly adequate model for proper characterization of the wind condition of the region. Various wind energy quantities which include the potential wind power and the wind speed with the highest amount of energy was computed based on the Weibull model. Results from the study further suggest that the region can get a fair share of energy generation from renewable energy sources such as the wind.
Keywords: maximum likelihood, Weibull distribution, wind speed, wind energy, wind power

Introduction
Wind is a natural phenomenon related to the movement of air masses influenced primarily by the differential solar heating of the earth (Sambo, 2005). Globally, two major factors drive large scale wind patterns (the atmospheric circulation); the differential heating between the equator and the poles (difference in absorption of solar energy to buoyancy forces) and the rotation of the planet. The study of wind pattern and distribution are often carried out on the speed of the wind in a given wind regime and in which case, wind speed is treated as a stochastic variable. Wind speed is capable of a generating output called *wind power* which is normally realized by employing wind energy conversion devices. Even though wind power is very consistent from year to year, it is also subject to significant variations over shorter time scale; hourly, daily or seasonally (Odo *et al.,* 2012). To optimize the design of a wind energy conversion device, data on wind speed range over which the device must operate to maximize energy extraction is required. This in turn requires the knowledge of the probability distribution of the wind speed. The 2-parameter Weibull distribution has been used extensively in the literature to model many wind regimes (see e.g. Slootweg et. al, 2001; Rehman *et al.,* 1994; Bivona *et al.,* 2003; Celik, 2004; Yilmaz and Celik, 2008; Sarkar and Kasperki, 2009; Zaharim *et al.,* 2009; Sarkar et al. 2011; Odo et al. 2012; Osatohanmwen *et al.,* 2017). This is due to the flexibility of the Weibull model in

capturing the wind distribution of many locations.

In this paper, a statistical assessment of the wind energy potential of south-eastern Nigeria is carried using monthly average wind speed observations obtained from the region for the period (1987 – 2019). The observations were obtained from a recording station in the city of Awka the capital of Anambra state. The rest of the paper is organized as follows: In section 2 the Weibull wind speed model and its properties are presented. The maximum likelihood method of estimation of the Weibull shape and scale parameters is contained in section 3. Some important wind energy quantities based on the Weibull model are presented in section 4. Section 5 contains analysis of data and results. The paper closes in section 6 with a conclusion.

Materials and Method

The Weibull wind speed model

The Weibull wind speed model is a continuous probability distribution with probability density function (pdf) and cumulative distribution function (cdf) given respectively by

$$f(x) = \frac{k}{c}\left(\frac{x}{c}\right)^{k-1} \exp\left(-\left(\frac{x}{c}\right)\right)^{9} \quad (1)$$

$$f(x) = 1 - \exp\left(-\left(\frac{x}{c}\right)^{k}\right) \quad (2)$$

X > 0,k,c,> 0

Where k and c are shape and scale parameters respectively. When applied to wind speed studies, the parameter k is a dimensionless shape parameter that shows the peakedness of the distribution of the wind speeds at the measuring location, and for varying values of k the distribution of the wind speeds takes the shape of other distributions. For k = 1, the distribution is exponential, for k = 2 it is Rayleigh distribution, and for k = 3.4, the distribution becomes approximately normal. The scale parameter

$$\mu_r' = \int_{-\infty}^{\infty} x^r f(x)dx, \qquad r = 1,2, \ldots \quad (3)$$

For the Weibull model, the r^{th} moment about the origin is given by

$$\mu_r' = \int_0^{\infty} x^r \frac{k}{c}\left(\frac{x}{c}\right)^{k-1} \exp\left(-\left(\frac{x}{c}\right)^k\right) dx \;\; r = 1,2, \ldots \quad (4)$$

Evaluating (4) gives

$$\mu_r' = c^r \Gamma\left(1 + \frac{r}{k}\right), \quad (5)$$

Where Γ (.) is the complete gamma function defined as

$$\Gamma(n) = \int_0^{\infty} x^{n-1} e^{-x} dx.$$

The mean of the Weibull random variable *X* is obtained from (5) by setting r = 1 to obtain

$$\mu_1' = c\,\Gamma\left(1 + \frac{1}{k}\right). \quad (6)$$

The variance defined as $\sigma^2 = \mu_2' - (\mu_1')^2$ for the Weibull distribution is thus given as

$$\sigma^2 = c^2\left[\Gamma\left(1 + \frac{2}{k}\right) - \Gamma^2\left(1 + \frac{1}{k}\right)\right]. \quad (7)$$

Thus, the average wind speed of a particular wind regime using the Weibull model can be obtained using (6). The dispersion of the wind condition can also be obtained from (7).

For the Weibull distribution, the mode is obtained by differentiating the pdf in (1) w.r.t

$$f'(x) = \frac{k(k-1)}{c^2}\left(\frac{x}{c}\right)^{k-2} \exp\left[-\left(\frac{x}{c}\right)^k\right] - \left(\frac{k}{c}\right)^2 \left(\frac{x}{c}\right)^{2(k-1)} \exp\left[-\left(\frac{x}{c}\right)^k\right].$$

Setting the above expression to zero and solving for x produces the mode of the Weibull distribution given by

$$X_{mp} = c\left(\frac{k-1}{k}\right)^{1/k} \quad (8)$$

Using the Weibull model, (8) is used to determine the most probable wind speed of a given wind regime.

Maximum likelihood estimation of the Weibull shape and scale parameters

Given the pdf *f (x)* of the Weibull distribution with parameters *k* and c, and a random independent sample of observations x_1, x_2, ………. X_n of size n, the maximum likelihood estimates of the parameters k and c are obtained by maximizing the log-likelihood function.

$$L = \sum_{i=1}^{n} \ln(f(x_i)). \quad (9)$$

For the Weibull distribution, the log-likelihood function is given as

$$L = n\ln k + (k-1)\sum_{i=1}^{n} \ln x_i - nk\ln c - \sum_{i=1}^{n}\left(\frac{x_i}{c}\right)^k \quad (10)$$

To obtain the estimate of k and c we take the partial derivative of (10) w.r.t. *k* and c and equate the resulting partial derivatives to zero. This gives

$$\frac{\partial L}{\partial k} = \frac{n}{k} + \sum_{i=1}^{n} \ln x_i - n\ln c - \sum_{i=1}^{n}\left(\frac{x_i}{c}\right)^k \ln\left(\frac{x_i}{c}\right) = 0, \quad (11)$$

$$\frac{\partial L}{\partial c} = \frac{k}{c}\sum_{i=1}^{n}\left(\frac{x_i}{c}\right)^k - \frac{nk}{c} = 0. \quad (12)$$

Observe that (11) and (12) are systems of non-linear equations in *k* and c. From (12) we obtain

$$c = \left(\frac{1}{n}\sum_{i=1}^{n} x_i^k\right)^{\frac{1}{k}}. \quad (13)$$

Substituting c in (13) into (11) and evaluating gives

$$\frac{1}{k} + \frac{1}{n}\sum_{i=1}^{n} \ln x_i - \frac{\sum_{i=1}^{n} x_i^k \ln x_i}{\sum_{i=1}^{n} x_i^k} = 0. \quad (14)$$

The system of equation in (14) is non-linear in *k*, the estimate k of k can be numerically determined by finding the root of the equation. Once k is obtained the estimate ĉ of c is then calculated from (13) as

$$\hat{c} = \left(\frac{1}{n}\sum_{i=1}^{n} x_i^{\hat{k}}\right)^{\frac{1}{\hat{k}}}. \quad (15)$$

The Weibull model in wind power analysis

Usually, before the design of wind energy conversion systems, analysis of wind characteristic of any location is needed in order to optimize energy output from such systems. Obviously, the wind speed presents the most important aspect of wind resources. Once a probability distribution have been chosen to model the wind speed, several characteristics of the wind speed is determined, and the design, analysis and assessment of power output of a turbine is ascertained.

Wind speed as air in motion possesses kinetic energy. For a given air mass

$$K.E = {}^1/_2\, m x^2 \quad (16)$$

To obtain the mass *m* of flowing air passing through an area *A* perpendicular to its velocity, its volume after time *t* has elapsed is multiplied by the air density ρ (whose value is normally taken to be 1.225kg /m^3) this gives the value of m as

$$m = Ax\rho t \quad (17)$$

Putting (17) into (16) gives the total wind energy as

$$E_x = ½ A\rho x^3 t \quad (18)$$

To obtain the total wind power, we consider that power is the rate of change of energy with time. Thus we differentiate (18) with respect to

$$P_x = ½ A\rho x^3 \quad (19)$$

Observe that the wind power is proportional to the cube of the wind velocity x. The actual average wind power based on a sample of wind speeds observations is obtained from (19) by defining x^3 as the mean value of the third power of the wind speeds observations. It follows that the average wind power (WP) associated with a pdf *f(x)* of the wind speed random variable *X* is given as

$$WP = \int_{-\infty}^{\infty} P_x f(x)dx. \quad (20)$$

Observe also that the average WP given by (20) is obtained by computing the third moment about zero of the wind speed random variable *X*. If *X* follows the Weibull distribution, one readily obtain the WP using (5) as

$$WP = 0.6125c^3\Gamma\left(1 + \frac{3}{k}\right). \quad (21)$$

Hence, the wind power output of a given wind regime can be calculated using (21) if the distribution of the regime's wind speeds approximates or follows the Weibull distribution.

When harnessing wind energy for power generation, one crucial aspect to deal with is the choice of the wind turbine. This choice is made based on the wind profile of the generating site. Two important parameters that determine the operation of the wind turbine is the modal or the most probable wind speed X_{mp} and the wind speed carrying the maximum energy X_{max}. The wind speed carrying the maximum energy X_{max}. using the Weibull distribution, as given in Nigim and Parker (2007) is

$$X_{max} = c\left(\frac{k+2}{k}\right)^{1/k} \qquad (22)$$

Where c and k are the Weibull scale and shape parameters respectively.

Data

For the analysis, monthly average wind speed observations obtained from a recording station in south-eastern Nigeria is used. The observations are for the period January 1987 – September 2019 spanning 393 months. The minimum and maximum wind speed from the data is respectively 0.7m/s and 5.8m/s. The mean wind speed from the data is 3.6m/s. A time series plot of the data set is given in Figure 1. The wind speed observations can be made available upon request from the corresponding author

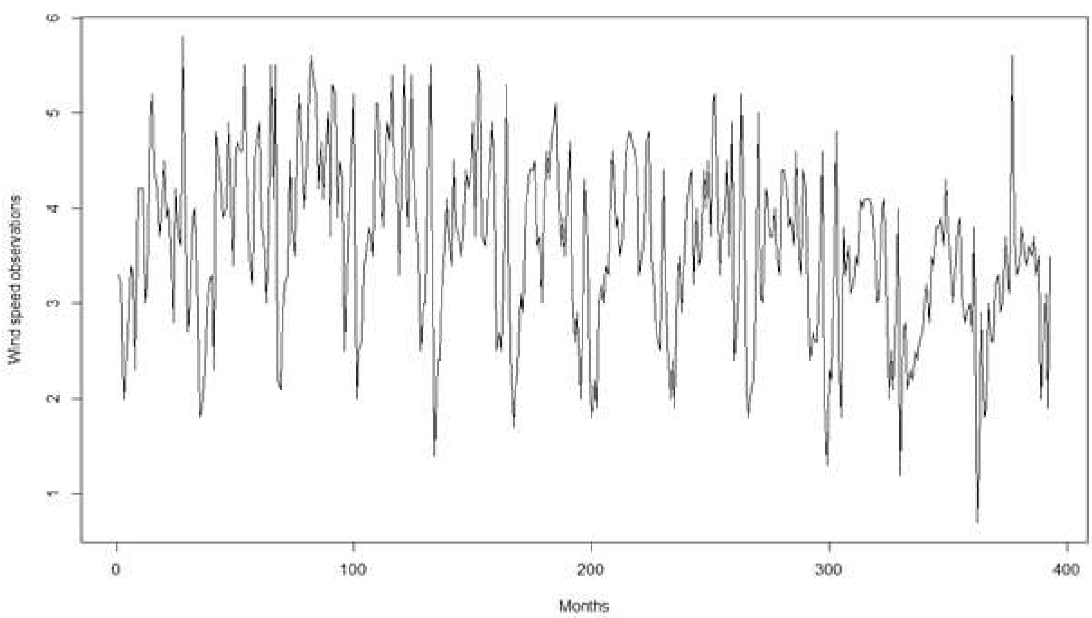

Figure 1: Time series plot of wind speed observations

Result and Discussion

The Weibull distribution is used to fit the wind speed observations using the maximum likelihood method and result of the parameter estimates of the distribution based on the sample of observations are contained in Table 1 alongside the value of the Kolmogorov – Smirnov (K-S) statistic and its corresponding p-value. The p-value of the K-S statistics is used to test if the wind speed observations can be modeled using a chosen distribution function (in our case, the Weibull distribution function). The graphs of the density function, cdf, Q – Q plot and the P – P plots of the fitted Weibull distribution is also given in Figure 2 (a-d). Useful values for the wind power of the region are reported in Table 2.

Table 1: Maximum Likelihood estimates of the Weibull shape and scale parameters

Weibull parameter estimates	$\hat{c} = 3.9547$ $\hat{k} = 4.3570$
K-S(p -value)	0.03712 (0.6372)

Results in Table 1 clearly show that the Weibull distribution is a very good model for modeling the wind regime. This is because the p-value of the K-S statistic is higher than 0.05 which is the level of significance for testing the hypothesis that the distribution of the wind speed is the same as the Weibull distribution. The adequacy of the Weibull distribution in capturing the behavior of the wind regime is also emphasized by the fact that the estimated density, cumulative probability, theoretical quantiles and theoretical probabilities neatly fits the histogram, empirical cumulative probability, empirical quantiles and the empirical probabilities respectively as shown in Figure 2(a-d).

Table 2: Weibull wind power quantities and their values

Quantity	Value
Average wind speed	3.60022m/s
Most probable wind speed	3.7250m/s
Wind speed with maximum energy	4.3129m/s
Wind power output	34.3418W/m^2
Wind speed standard deviation	0.9353m/s

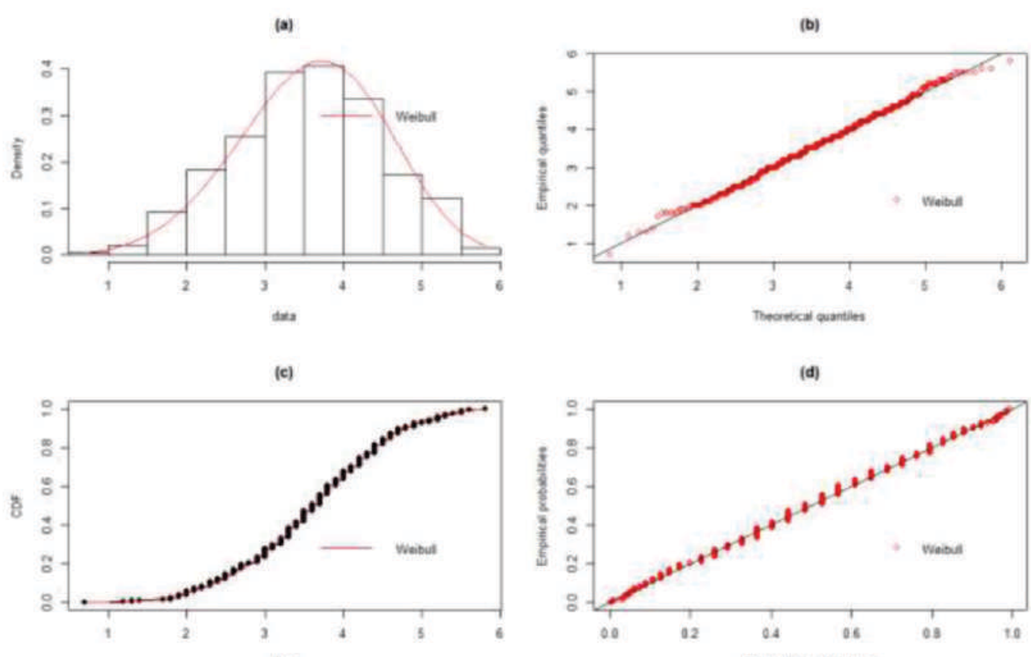

Figure 2 (a-d)

From Table 2, one observes that the Weibull model based average wind speed for the regime is 3.60022m/s which is almost the same as that obtained from the sample. This further confirms the validity of the Weibull model as a very suitable model for capturing the wind behavior of the location. Table 2 further reveals that south-eastern Nigeria is capable of generating a wind power of 34.3418W/m^2 of wind energy, a result which is very encouraging and beckons the interest of State Actors to look into wind power generation in the region.

In conclusion, an assessment of the wind energy potential of south-eastern Nigeria has been carried out in this paper and results have indicated the suitability of the Weibull model in capturing the wind regime of the location. Further result from the study revealed that the region is capable of producing 34.3418W/m^2 of wind energy, a situation which beckons the attention of government and private individuals to invest in the development of wind power generation in the region.

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