



MATHEMATICAL MODEL OF THE UPHEAVAL/SUBSIDENCE PHENOMENON OF THE SOIL UNDERNEATH STRUCTURES

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Abstract

Degradation models used to predict the future state of components often involve simplifications and assumptions to compensate a lack of data, imprecision and vagueness, which cannot be ignored. To overcome these issues, the imprecise probabilities framework and Markovian approach are proposed for performing reliability analysis, decision-making, and risk-based design and maintenance. The reliability assessment is performed by considering the simultaneous action of many natural and technological loads, which are random by nature and can be adequately described only by stochastic processes which are not performed due to lack of valid calculation methods. This methodology has been applied to study the reliability of arctic pipeline infrastructure. It is found that the process of heaving-subsidence support can be considered as two of the Markov process: when frost heave - pure birth Markov process; and at the subsidence of support - pure death Markov process. This is applicable to soils underneath foundations for structures and infrastructures.

Keywords: Mathematical model; Upheave and subsidence; Structures; Markovian process; Loads; Reliability

Introduction

The actual loads and impacts in the form of random variables are rare and can be considered as a certain idealization of the actual loads. Many wearing impacts (friction, erosion, cavitation, corrosion and so on) and shock loads are adequately described by the model of the second group. Meteorological loads - snow, wind (static component) is described by the model of discrete Markov processes, diffusion and semi-Markov models (see e.g., Gnedenko et al., 1965; Barucha-Rheid, 1969; Timashev, 1982). Loads of near and far acoustic field, atmospheric turbulence and turbulence in

the boundary layer, wave pressure and seismic impacts are most aptly described by the differentiable random processes and fields (the latter in conjunction with the second group of models).

Loads and impacts acting on the arctic pipelines generally are random processes. To solve the problems of arctic pipelines reliability under the action of combinations of random loads and impacts, one needs to have their models as initial data. Adequate interpretation of these loads is possible in different ways, depending on the type and degree of completeness of primary statistical data, aims and objectives of research and the

required presentation forms of the results. In solving the problems of arctic pipelines reliability, we need: (Timashev, 1982; Timashev, et al., 2016; Opeyemi et al., 2015a, 2015b and 2016, Burukhina et al., 2021):

- load models as random processes, which adequately account for all their basic physical properties; and
- possibility to calculate easily enough the probability of staying of these loads on arbitrary, including low levels.

The development of any stochastic model for a real process is always a compromise between the desired level of detail describing the process and feasibility of achieving it. One of the simplest and at the same time available model for the description of such

processes is Markov process of the “birth and death” type.

The reliability of the arctic pipeline operating under impact of two loads from subsidence/frost upheaval of support and from corrosion defects is presented in this paper.

Materials and Methods

Stochastic model on loads and impacts acting on arctic pipelines

Consider one calendar year as a cycle of process of subsidence-heave support. Conditionally, this cycle can be divided into the two periods: winter (when the process of frost heaving) and summer (the period when the process of support subsidence on seasonal thawing soils). This is visually it is demonstrated in Fig. 1.

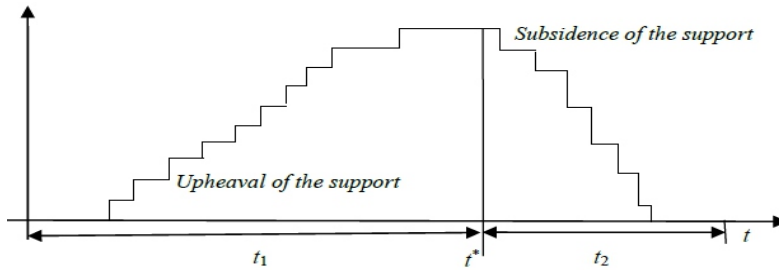


Figure 1: One cycle of upheaval-subsidence process of arctic pipeline support

Obviously, the time period for winter t_1 and for summer t_2 and time t^* are random variables. Moreover, $T = t_1 + t_2 = 365$ days. Thus, the process of heaving-subsidence support can be considered as two of the Markov process: when frost heave - pure

birth Markov process; at the subsidence of support - pure death Markov process.

In fact, there is a case where the support subsidence occurs by an amount greater than the one which started the process of frost heaving. This case is shown in Fig. 2.

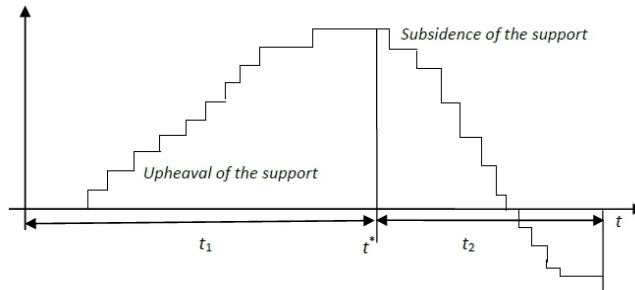


Figure 2: One cycle of upheaval-subsidence of the support process for the case where the value of subsidence of the support does not match the magnitude with which began the process of frost upheaval.

In this case, the process is below the level of the beginning of the process of frost heaving and must be considered separately. In the next cycle, the process of frost heaving begins with level of subsidence support.

We divide the range of the possible values of the considered load on M disjoint intervals (states). If the possible values of the load on the arctic pipeline can only increase or decrease in time, and at random time moments can transit from the i -th state only to the $(i + 1)$ -th state or $(i - 1)$ -th, then such a transition process can be described by a pure birth or death Markov process.

The system of differential equations that describes this process is

$$\begin{cases} \frac{dP_1(t)}{dt} = -\lambda_1 P_1(t); \\ \frac{dP_i(t)}{dt} = \lambda_{i-1} P_{i-1}(t) - \lambda_i P_i(t), \quad i = 2, \dots, M-1; \\ \frac{dP_M(t)}{dt} = \lambda_{M-1} P_{M-1}(t), \end{cases} \quad (1)$$

where $P_i(t)$ is the probability that the load value is in the i -th state at the moment of time t , $\lambda_i(t)$ is the transition intensity form i -th state to the $(i + 1)$ -th state. For pure death process the intensity $\lambda_i(t)$ replaced by the transition intensity $\mu_i(t)$ from i -th state to the $(i - 1)$ -th state.

If at the initial time t_0 load value is in a first state, then the solution of system of differential equations in Eq. (1) has the form:

$$P_i(t) = \sum_{j=1}^i \eta_{ij} \exp[-\lambda_j t], \quad i = 1, 2, \dots, M, \quad (2)$$

where:

$$\eta_{ij} = \begin{cases} \eta_{i1} = 1; \\ \eta_{i-1,j} \cdot \frac{\lambda_{i-1}}{\lambda_i - \lambda_j}, \quad j \neq i, i = 2, \dots, M, j = 1, 2, \dots, (i-1); \\ -\sum_{q=1}^{i-1} \eta_{iq}, \quad j = i, i = 2, \dots, M. \end{cases} \quad (3)$$

The influence of the arctic pipeline defects on failure (burst) pressure as a

homogeneous pure death Markov process can be described as follows.

Divide the possible range of change of the burst pressure of a pipeline defective cross section $(P_{op}; P_f(0))$ into $M-1$ non-overlapping equal interval $I_i (i=M-1, \dots, 1)$. Here $P_f(0)$ is the defect failure pressure at initial time $t = 0$. The last interval (conditional failure state) I_M which includes the lowest values of failure pressure is taken as $(0; P_{op})$.

The system of differential equations, that describes this process, has the form

$$\begin{cases} \frac{dP_1(t)}{dt} = -\mu_1(t) P_1(t), \\ \frac{dP_i(t)}{dt} = \mu_{i-1}(t) P_{i-1}(t) - \mu_i(t) P_i(t), \quad (i = 2, \dots, M-1), \\ \frac{dP_M(t)}{dt} = \mu_{M-1}(t) P_{M-1}(t), \end{cases} \quad (4)$$

where $P_i(t)$ is the probability that the failure (burst) pressure $P_f(t)$ of defective cross section is in the i -th state at time t , $\mu_i(t)$ is the intensity of transition from the i -th state to the $(i + 1)$ -th. The quantity $\mu(t)$ may be associated with the rate of change of random variables $P_f(t)$ as follows:

$$\mu(t) = -\frac{P_f'(t)}{\Delta I}, \quad (5)$$

Where ΔI is the interval length, P_f' is the derivative of the function $P_f(t)$ with respect to time at time t . The minus sign in this formula is because the derivative of monotonously decreasing function has negative values in the whole domain of its definition.

Now the system of differential equations in Eq. (4) can be rewritten as

$$\begin{cases} \frac{dP_1(t)}{dt} = -\mu(t) P_1(t), \\ \frac{dP_i(t)}{dt} = \mu(t) P_{i-1}(t) - \mu(t) P_i(t), \quad (i = 2, \dots, M-1), \\ \frac{dP_M(t)}{dt} = \mu(t) P_{M-1}(t), \end{cases} \quad (6)$$

It is obvious that at the initial moment of time $t = 0$ the random variables $P_f(0) \in I_i$. Hence, the initial conditions for the system of differential

equations in Eq. (6) will be:

$$P_i(0) = 1, P_i(0) = 0, (i=1, \dots, M)$$

The general solution of system of differential equations in Eq. (6) will be as follows:

$$\begin{cases} P_i(t) = \frac{\rho^{i-1}(t)}{(i-1)!} \cdot \exp\{-\rho(t)\}, i=1, \dots, M-1, \\ P_M(t) = 1 - \left[\exp\{-\rho(t)\} + \sum_{i=2}^{M-1} \frac{\rho^{i-1}(t)}{(i-1)!} \cdot \exp\{-\rho(t)\} \right], \end{cases} \quad (7)$$

Where $P_i(t)$ is the probability that the failure (burst) pressure of defective cross section is in the i -th state at the moment of time t , $\rho(t)$ is calculated using formula:

$$\rho(t) = \int_0^t \mu(\tau) d\tau - \int_0^t \frac{P_f'(\tau)}{\Delta} d\tau = \frac{P_f(t) - P_f(0)}{\Delta}. \quad (8)$$

An explicit description of the mathematical model proposed for use in the upheaval/subsidence phenomenon of the soil underneath structures/infrastructures has been outlined and explained in detail here. This numerical modelling which employs the theoretical and computational frameworks of Markovian approaches forms the major approaches for the solutions to the problem statement of the need for reliability and maintenance of structures under severe uncertainty as a key issue in ensuring a faultless life of engineering structures and systems despite fluctuations and changes of structural and environmental parameters and conditions.

Example Applications

Consider the calculation of the reliability of the arctic pipeline where it operates under impact of two loads from subsidence / frost upheaval of support and from corrosion defects. The model of loads and impacts acting on arctic pipelines is described above in materials and methods section.

Assume that the load of the subsidence (upheaval) of the support is described by pure death Markov process or pure birth Markov process $q_1(t)$ with intensities of transition $\lambda_i, I = 1, 2, \dots, M_1$, and load from the arctic pipeline defectiveness - by means

pure death Markov process $q_2(t)$ with intensities of transition $\mu_j, j = 1, 2, \dots, M_2$; moreover, $q_1(t), q_2(t)$ are independent processes. Refer to the model on loads and impacts acting on arctic pipelines in section earlier outlined.

Let $z(t) = \{q_1(t), q_2(t)\}$ be a two-dimensional process. Then the system of differential equations of this process is:

$$\begin{cases} \frac{dP_{1,1}(t)}{dt} = -(\lambda_1 + \mu_1)P_{1,1}(t); \\ \frac{dP_{i,j}(t)}{dt} = -(\lambda_i + \mu_j)P_{i,j}(t) + \lambda_{i-1}P_{i-1,j}(t) + \mu_{j-1}P_{i,j-1}(t), i, j = 2, 3, \dots \end{cases} \quad (9)$$

The relative probabilities are:

$$P_{i,j}(t) = P\{q_1(t)=i, q_2(t)=j\} \quad (10)$$

The initial conditions for the system of differential equations in Equation (9) have the form

$$P_{i,j}(t) = 1, P_{i,j}(t) = 0, i, j = 2, 3, \dots$$

If we select an area Ω in space with boundary Γ and introduce the auxiliary process $\bar{Z}(t)$ so that $\bar{\lambda}_i = \lambda_i, \bar{\mu}_j = \mu_j$ If the point $(i,j) \in \Omega$ and $\bar{\lambda}_i = 0, \bar{\mu}_j = 0$ if $(i,j) \in \Gamma$ that is, the boundary is absorbing. Then the probability of non-way out of the process $Z(t)$ from area Ω will be calculated by the formula

$$P(t) = \sum_{i=1}^{m_1-1} \sum_{j=1}^{n_1-1} \bar{P}_{i,j}(t), \quad (11)$$

Where $\bar{P}_{i,j}(t)$ satisfies the following system of differential equations:

$$\begin{cases} \frac{d\bar{P}_{1,1}(t)}{dt} = -(\bar{\lambda}_1 + \bar{\mu}_1)\bar{P}_{1,1}(t); \\ \frac{d\bar{P}_{i,j}(t)}{dt} = -(\bar{\lambda}_i + \bar{\mu}_j)\bar{P}_{i,j}(t) + \bar{\lambda}_{i-1}\bar{P}_{i-1,j}(t) + \bar{\mu}_{j-1}\bar{P}_{i,j-1}(t), i, j = 2, 3, \dots \end{cases} \quad (12)$$

Equation (11) reflects the fact that the probability of non-way out of the process $Z(t)$ from area Ω is equal to the probability of finding the process $\bar{Z}(t)$ inside the area Ω at time t .

The probability estimation using Eq. (11) is the true function of arctic pipeline reliability at influence on its two considered loads.

Considering the probability, we construct a

two-sided estimate of this function:

$$P_i^{(m_i)}(t), i = 1, 2, \dots, m_j; P_j^{(n_j)}(t), j = 1, 2, \dots, n_i; \quad (13)$$

This is the expression of the probability that at time t the process $q_1(t)$ [$q_2(t)$] is in a state $i(j)$ on the condition that the state m_j (n_i) is absorbing. These probabilities are determined by solving the following system of differential equations:

$$\begin{cases} \frac{dP_i(t)}{dt} = -\lambda_i P_i(t); \\ \frac{dP_i(t)}{dt} = \lambda_{i-1} P_{i-1}(t) - \lambda_i P_i(t), \quad i = 2, \dots, m_j - 1; \\ \frac{dP_{m_j}(t)}{dt} = \lambda_{m_j-1} P_{m_j-1}(t); \\ P_i(0) = 1, \quad P_i(0) = 0, \quad i = 2, \dots, m_j. \end{cases} \quad (14)$$

$$\begin{cases} \frac{dP_i(t)}{dt} = -\mu_i P_i(t); \\ \frac{dP_j(t)}{dt} = \mu_{j-1} P_{j-1}(t) - \mu_j P_j(t), \quad j = 2, \dots, n_i - 1; \\ \frac{dP_{n_i}(t)}{dt} = \mu_{n_i-1} P_{n_i-1}(t); \\ P_i(0) = 1, \quad P_j(0) = 0, \quad j = 2, \dots, n_i. \end{cases}$$

The solutions to these systems of differential equations are determined from the Equation (15) or (17).

$$P_i(t) = \sum_{j=1}^i \eta_{ij} \exp[-\lambda_j t], \quad i = 1, 2, \dots, M, \quad (15)$$

where:

$$\eta_{ij} = \begin{cases} \eta_{11} = 1; \\ \eta_{i-1,j} \cdot \frac{\lambda_{i-1}}{\lambda_i - \lambda_j}, \quad j \neq i, i = 2, \dots, M, j = 1, 2, \dots, (i-1); \\ -\sum_{q=1}^{i-1} \eta_{iq}, \quad j = i, i = 2, \dots, M, \end{cases} \quad (16)$$

$$\begin{cases} P_i(t) = \frac{\rho^{i-1}(t)}{(i-1)!} \cdot \exp\{-\rho(t)\}, \quad i = 1, \dots, M-1, \\ P_M(t) = 1 - \left[\exp\{-\rho(t)\} + \sum_{i=1}^{M-1} \frac{\rho^{i-1}(t)}{(i-1)!} \cdot \exp\{-\rho(t)\} \right], \end{cases} \quad (17)$$

Where $P_i(t)$ is the probability that the burst pressure (BP) of defective cross section is in the i -th state at the moment of time t , $\rho(t)$ is calculated using formula:

$$\rho(t) = \int_0^t \mu(\tau) d\tau - \int_0^t \frac{P_f'(\tau)}{\Delta I} d\tau = \frac{P_f(t) - P_f(0)}{\Delta I}. \quad (18)$$

The two-sided estimate of the true reliability function $R(t)$ of arctic pipeline at the combination of the two loads is given by (see e.g., Timashev, 1982):

$$R_1 \leq R(t) \leq R_2(t), \quad (19)$$

where:

$$\begin{aligned} R_1(t) &= \sum_{i=1}^{m_j-1} \sum_{j=1}^{n_i-1} P_i^{(m_i)}(t) P_j^{(n_j)}(t), \\ R_2(t) &= \sum_{i=1}^{m_j-1} \sum_{j=1}^{n_i-1} P_i^{(m_i)}(t) P_j^{(n_j)}(t). \end{aligned} \quad (20)$$

We have plotted the marginal wind velocity on the operating pressure (see Fig. 3) and calculate the ultimate values for wind speed using Equation (21) and (22).

$$q_w = (q_n^c + q_n^d) D_{in}, \quad (21)$$

Where q_n^c is the normative value of the static component of wind load, N/m^2 , determined according to (SP 20, 2011); q_n^d is the normative value of the dynamic component of wind load (N/m^2), determined according to (SP 20, 2011) as well as for buildings with a uniformly distributed mass and constant stiffness; and D_{in} is outer pipeline diameter, m, with the insulating cover and the lining.

$$q_n^c = w_0 k(Z_e) c, \quad (22)$$

Where w_0 is the normative value of wind pressure; $k(Z_e)$ is the coefficient that considers the change of wind pressure at height Z_e ; and c is the aerodynamic coefficient.

For simplicity, we do not consider the dynamic component of wind load. Consider section of the arctic pipeline which is 2 m above the ground, and the type of terrain is A. The equivalent height $z_e = 2 + 0.350/2 = 2.175$ m. From:

$$K(Z_e) = 1.0 \cdot (2.175/10)^{2.015} = 0.633.$$

Aerodynamic coefficient $c = 0.5$. From Equation (21) without considering the dynamic component, it follows that

$$q_w = 0.43 v_{50}^2 k(Z_e) c D_{in}, \quad (23)$$

It can be deduced from this Equation (21) that

the wind speed that can occur once in 50 years could be estimated by:

$$V_{50} = \sqrt{\frac{q_w}{0.43k(Z_e)cD_{in}}} \tag{24}$$

Let time $t = 10$ years and the operating

pressure $P_{op} = 5.4$ MPa. Substituting q_w into the Equation (23) from Equation (24) we obtain the ultimate limit values of wind load, and the ultimate permissible wind speed values. The results are shown in Fig. 3 and 4.

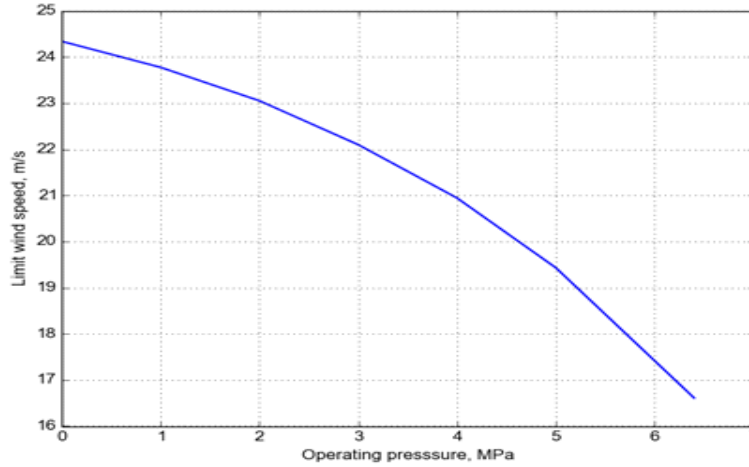


Figure 3: Ultimate permissible wind speed at time $t = 10$ years, depending on the operating pressure

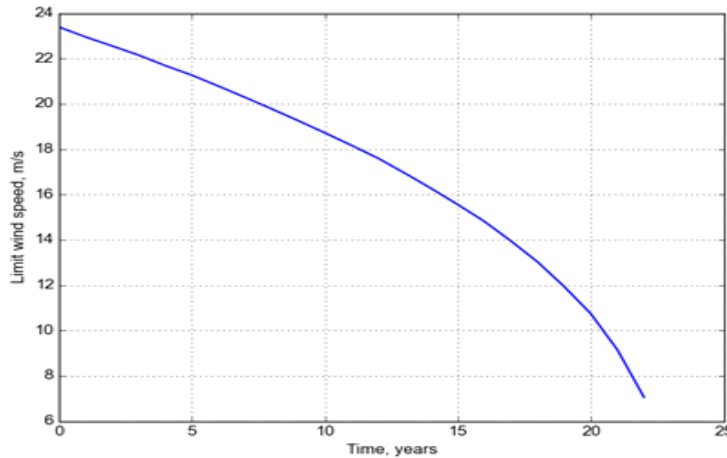


Figure 4: Ultimate permissible wind speed at operating pressure $P_{op} = 5.4$ MPa, depending on the time (corrosion rate)

In accordance with Fig. 4, at $t = 10$ years and $P_{op} = 5.4$ MPa, the ultimate wind speed is equal to 18.7 m/sec. The interval probability of occurrence of such wind speed value is equal to [0.64; 0.95]. Hence, the point wise pipeline reliability (Rpl) in this case will be $0.64 \leq Rpl \leq 0.95$. Integrating the whole curve of Fig. 4, gives the overall interval of pipeline reliability.

Considering the limiting wind speed as a random discrete value, we can build a cumulative distribution function, i.e., the probability of failure against limit wind speed. The resulting function is shown in Fig. 5. Two-sided reliability assessment of pipeline probability of failure depending on the limit wind speed is shown in Figs. 6 and 7.

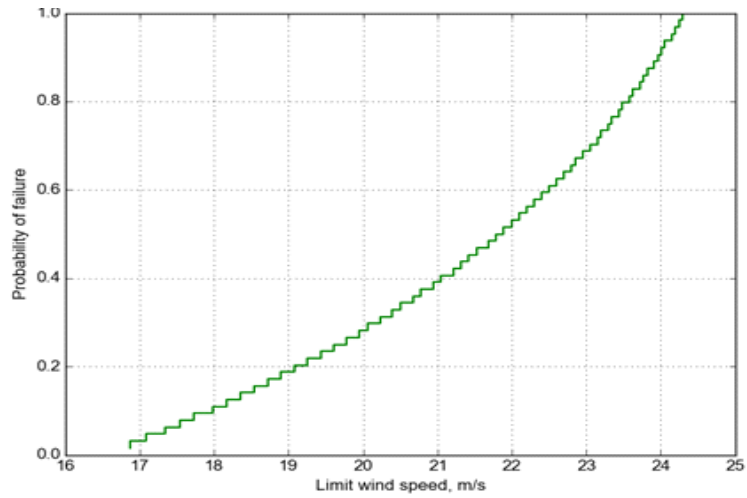


Figure 5: The probability of failure of the pipeline depending on the wind speed limit

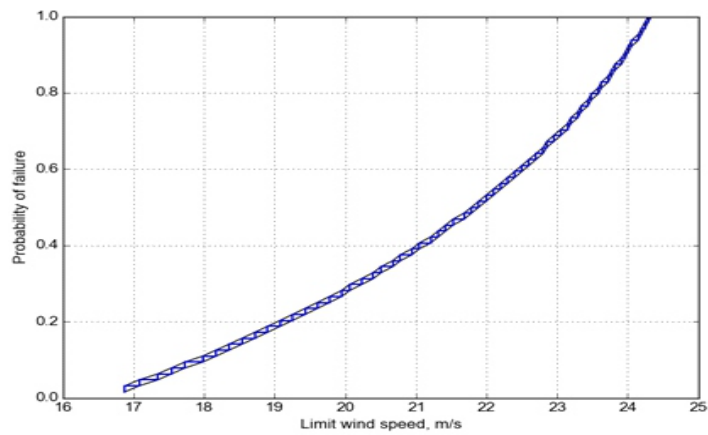


Figure 6: Two-sided reliability assessment of pipeline failure probability depending on the limit wind speed

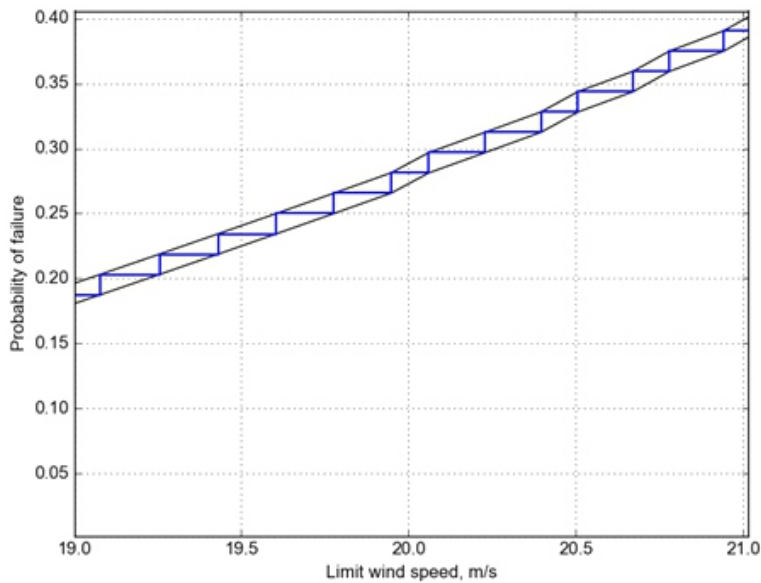


Figure 7: Two-sided reliability assessment of pipeline failure probability depending on the limit wind speed from 19 to 21 m/s (magnified).

Conclusion

The reliability of the arctic pipeline operating under impact of two loads from subsidence/frost upheaval of support and from corrosion defects is presented in this paper. It is found that the process of heaving-subsidence support can be considered as two of the Markov process: when frost heave - pure birth Markov process; at the subsidence of support - pure death Markov process. The advantage of this approach is that it is applicable and suitable for use in soils underneath foundations (such as pad, strip, etc.) for structures and infrastructures. The risk associated with ignoring of subsidence of soil underneath structure is that the process will cause the walls and floors (e.g. in buildings) to shift, leading to cracks and potentially destabilizing the construction of the property.

Acknowledgement

This work has been supported by the Science & Engineering Center "Reliability and Safety of Large Systems and Machines" Ural Branch Russian Academy of Sciences, Ekaterinburg, Russia, under the leadership of Prof. S. A. Timashev.

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