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DYNAMICS OF BEAMS ON FILONENKO-BORODICH FOUNDATION

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Abstract
This paper is concerned with the dynamic analysis of beams resting on Filonenko-Borodich foundation. The fourth order partial differential equations governing the motion of the Rayleigh beams are transformed into a set of ordinary differential equations that are eventually solved using Galerkin's method in conjunction with Laplace and convolution theory. Numerical results in plotted curves were presented to determine the vibration characteristics of some pertinent structural parameters. The study also analyses the circumstances under which resonance occur in the dynamical system.

Keywords: Rayleigh Beam, Filonenko-Borodich Foundation, Garlekin Method, Resonance, Foundation Stiffness.

Nomenclature

$E\,[N/m^2]$: Modulus of Elasticity
$I[m^4]$: Moment of Inertia of the Beam
$k[N/m^3]$: Foundation Stiffness
$L[m]$: Length of the Beam
$M\,[kg]$: Mass of the Moving Load
$T\,[N]$: Tensile Force
$t\,[s]$: Time Coordinate
$u\,[m/s]$: Constant Velocity
$\varphi\,(x,t)[m]$: Deflection from the Equilibrium
$x[m]$: The Spatial Coordinates
$\mu[kg/m]$: Mass of the Beam per Unit Length

Introduction

In modern design, analysis of beams on elastic foundation is developed on the assumption that the reaction forces of the foundation are proportional at every point to the deflection of the beam at that point. The vertical deformation characteristics of the foundation are defined by means of identical, independent, closely spaced discrete and linearly elastic springs. The constant of proportionality of these springs is known as subgrade reaction coefficient, k. This simple and relatively crude mechanical representation of foundation was firstly introduced by Winkler in 1867, Hetenyi (1946), Kerr (1964). The concept of beams resting on Winkler elastic foundation has been an important tool for the modelling and analysis of structural highway, railroad engineering problems and extensive research in this area have been reported in literature Ding (1993), Oni (1996) and Ogunbamike (2012). Other researchers who worked on one dimensional solid include Holl (1950), Kenny (1954), Steel (1971), (Dasa et al, 2007), Akour (2010) and Bakhshi (2018) just to mention but a few.

Recently, the flexural vibration of a simply supported plate under travelling distributed loads was carried out by (Andi and Oni, 2014). Both the gravity and inertia effects of the loads are taken into consideration and the solution technique is based on the two-dimensional finite Fourier sine integral transformation and a modification of the Struble's asymptotic technique. It is clearly seen from the results that the response amplitude of the moving distributed mass system is higher than that of the moving distributed force system for fixed values of rotatory inertia correction factor and foundation moduli. Very recently, the transverse motions of Bernoulli-Euler beam resting on elastic foundation under two concentrated moving loads were considered by Adedowole (2019). In his work, the principal equation is the fourth order partial differential equation and the solution techniques are based on the Fourier sine transformation. I want to remark at this juncture that, in all the aforementioned works the problem of assessing the dynamic behaviour of beams carrying moving loads have almost exclusively been reserved in literature for moving loads resting on Winkler foundation Usman (2003) and (Awodola and Oni, 2013). However, the Winkler model, which has been originally developed for the analysis of railroad tracks, is very simple but does not accurately represent the characteristics of practical foundation. The model represents the subgrade as a system of closely spaced but mutually independent linear springs with the foundation reaction assumed to be proportional to the vertical displacement of the foundation. In addition, it predicts the discontinuous behaviour of the surface displacements beyond the load, in reality the surface displacement continues beyond the load (force) region. Historically, in order to overcome the deficiency of Winkler model and improve the model, a more realistic elastic model is proposed in 1940 by Filonenko and Borodich and is termed Filonenko-Borodich foundation Kerr (1964). This improved foundation model takes into account the effect of shear interaction among adjacent points in the foundation. In the model, the first parameter represents the stiffness of the vertical spring, as in the Winkler model, whereas the second parameter is introduced to account for the coupling effect of the linear elastic springs.

Problem Statement

The beam is resting on Filonenko-Borodich foundation and is subjected to the following conditions:

1. The beam material properties are linear.
2. The damping (c^0) and the stiffness (k) of the foundation are linear.
3. The tensile force (T) of the elastic springs is also linear.
4. The beam is prismatic.
5. The beam is simply supported (pin-pin ends)
6. The load applied is harmonic and distributed over the length of the beam.

Governing Equation

The governing equation of motion of a uniform Rayleigh beam resting on Filonenko-Borodich foundation and traversing by moving load with constant velocity is given by

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2\varphi(x,t)}{\partial x^2}\right)+\overline{m}\frac{\partial^2\varphi(x,t)}{\partial t^2}+c^0\frac{\partial\varphi(x,t)}{\partial t}-\mu r^0\frac{\partial^4\varphi(x,t)}{\partial x^2\partial t^2}+k\varphi(x,t)-T\frac{\partial^2\varphi(x,t)}{\partial x^2}=P\sin\lambda t\,\delta(x-ut) \quad \text{..... (2.1)}$$

In this study, it is assumed that the beam model is taken to be simply supported, that is the beam has simple supports at the end $x=0$ and $x=L$, hence the boundary condition takes the form

$$\varphi(0,t) = \varphi(L,t) = 0 \quad \ldots (2.2)$$

$$\frac{\partial^2 \varphi(0,t)}{\partial x^2} = \frac{\partial^2 \varphi(L,t)}{\partial x^2} = 0 \quad (2.3)$$

and the initial condition the beam is taken as

$$\varphi(x,0) = \frac{\partial \varphi(x,0)}{\partial t} = 0 \quad (2.4)$$

The travelling time t of the moving load is assumed to be limited to that interval of time within the moving load on the beam i.e

$0 \le ut \le L$ (2.5)

Galerkin's Method

This is one of the best methods used in solving problems involving mechanical vibrations. The Galerkin's method involves solving equation of the form

$L[\varphi] - P = 0$ (2.6)

where

L= the differential operator (linear or non-linear)

φ= the structural displacement

P = the transverse load acting on the structure.

A sequence of linearly independent function $Sin\frac{\pi x}{L}$ satisfying the boundary conditions are chosen and the approximate solutions sought in the form

$$\varphi(x,t) = \sum_{j=1}^{n} P_j(t) Sin\frac{j\pi x}{L} \quad (2.7)$$

Function P_j *(t)* is determined from the condition such that the expression should be orthogonal to the function $Sin\frac{\pi x}{L}$. In this way we get a set of ordinary differential equation

$$\int_0^L \left\{ L\left[\sum_{j=1}^{n} P_j(t) Sin\frac{j\pi x}{L}\right] - P \right\} Sin\frac{k\pi x}{L} dx = 0 \quad (2.8)$$

Approximate Analytical Solution

Substituting (2.7) into (2.1) and after some simplification and rearrangement yields

$$EI\left[\sum_{j=1}^{n} P_j(t)\left(\frac{j\pi}{L}\right)^4 + \overline{m}\ddot{P}_j(t) + c^0 \dot{P}_j(t)\right.$$

$$\left. -\mu r^0 P_j(t)\left(\frac{j\pi}{L}\right)^2 + kP_j(t) - TP_j(t)\left(\frac{j\pi}{L}\right)^2\right] Sin\frac{j\pi x}{L} = P\sin\lambda t\,\delta(x-ut) \quad (2.9)$$

The Galerkin's method requires that the RHS of (2.9) above becomes orthogonal to the function.

$Sin\frac{i\pi}{L}$ Multiply both sides of equation (2.9) by $Sin\frac{i\pi}{L}$ and integrating from 0 to *L,* we obtained

$$\int_0^L \sum_{j=1}^{n} \left\{ EIP_j(t)\left(\frac{j\pi}{L}\right)^4 + \overline{m}\ddot{P}_j(t) + c^0 \dot{P}_j(t) - \mu r^0 P_j(t)\left(\frac{j\pi}{L}\right)^2\right.$$

$$\left. + kP_j(t) - TP_j(t)\left(\frac{j\pi}{L}\right)^2\right\} Sin\frac{j\pi x}{L} Sin\frac{i\pi x}{L} dx = P\sin\lambda t\int_0^L \delta(x-ut) Sin\frac{i\pi x}{L} dx \quad (2.10)$$

Equation (2.10) can be rearrange to take the form

$$\sum_{j=1}^{n}\left(T_0 \ddot{P}_j(t) + T_1 \dot{P}_j(t) + T_2 P_j(t)\right) = P\sin\lambda t\int_0^L \delta(x-ut) Sin\frac{i\pi x}{L} dx \quad (2.11)$$

Where

$T_0 = d_0 + d_1$ (2.12)

$T_2 = d_2 + d_3 + d_4$ (2.13)

$$T_1 = c^0 \int_0^L Sin\frac{j\pi x}{L} Sin\frac{i\pi x}{L} dx \quad (2.14)$$

$$d_0 = \overline{m}\int_0^L Sin\frac{j\pi x}{L} Sin\frac{i\pi x}{L} dx \quad (2.15)$$

$$d_1 = \mu r^0 \int_0^L Sin\frac{j\pi x}{L} Sin\frac{i\pi x}{L} dx \quad (2.16)$$

$$d_2 = EI\left(\frac{j\pi}{L}\right)^4 \int_0^L Sin\frac{j\pi x}{L} Sin\frac{i\pi x}{L} dx \quad (2.17)$$

$$d_3 = k\int_0^L Sin\frac{j\pi x}{L} Sin\frac{i\pi x}{L} dx \quad (2.18)$$

$$d_4 = T\left(\frac{j\pi}{L}\right)^2 \int_0^L Sin\frac{j\pi x}{L} Sin\frac{i\pi x}{L} dx \quad (2.19)$$

Simplifying equation (2.11) further gives

$$\sum_{j=1}^{n}\left[\ddot{P}_j(t) + \Omega_0 \dot{P}_j(t) + \Omega_1 P_j(t)\right] = P_m Sin\lambda t Sin\phi t \quad (2.20)$$

where

$$\Omega_0 = \frac{T_1}{T_0},\quad \Omega_1 = \frac{T_2}{T_0},\quad P_m = \frac{P_0}{T_0} \quad \text{and} \quad \phi = \frac{i\pi c}{L} \quad (2.21)$$

Next, take the Laplace transformation of equation (2.20) described by

$$(\sim) = \int_0^\infty (\cdot) e^{-st} dt \quad (2.22)$$

Using Laplace transform in conjunction with the initial condition in equation (2.22), equation (2.11) becomes

$$[s^2+\Omega_0 s+\Omega_1]P_j(s)=\frac{P_m}{2}\left[\frac{s}{s^2+(\lambda-\phi)^2}-\frac{s}{s^2+(\lambda+\phi)^2}\right] \quad (2.23)$$

Equation (2.23) is transformed to give

$$P_j(s)=\frac{P_m}{2(\xi_1-\xi_2)}\left[\left(\frac{(\lambda-\phi)}{s^2+(\lambda-\phi)^2}\cdot\frac{1}{s-\xi_1}-\frac{(\lambda+\phi)}{s^2+(\lambda+\phi)^2}\cdot\frac{1}{s-\xi_1}\right)\right.$$
$$\left.-\left(\frac{(\lambda-\phi)}{s^2+(\lambda-\phi)^2}\cdot\frac{1}{s-\xi_2}-\frac{(\lambda+\phi)}{s^2+(\lambda+\phi)^2}\cdot\frac{1}{s-\xi_2}\right)\right] \quad (2.24)$$

where

$$\xi_1=\frac{-\Omega_0+\sqrt{\Omega_0^2-4\Omega_1}}{2} \text{ and } \xi_2=\frac{-\Omega_0-\sqrt{\Omega_0^2-4\Omega_1}}{2} \quad (2.25)$$

In order to obtain the Laplace inversion of equation (2.24), the following representation is adopted

$$f_1(s)=\frac{1}{s-\xi_1}, \qquad f_2(s)=\frac{1}{s-\xi_2}$$

$$g_1(s)=\frac{(\lambda-\phi)}{s^2+(\lambda-\phi)^2} \text{ and } g_2(s)=\frac{(\lambda+\phi)}{s^2+(\lambda+\phi)^2} \quad (2.26)$$

The Laplace inversion of each term of the right-hand side of equation (2.24) is their convolution. The convolution of f and g is denoted by

$$f*g=\int_0^t f(t-u)g(u)du \quad (2.27)$$

To this end, the Laplace inversion of equation (2.24) is easily expressed as

$$P_j(t)=P_c\left(e^{-\xi_1 u}I_1-e^{-\xi_2 u}I_2\right) \quad (2.28)$$

where

$$P_c=\frac{P_m}{2(\xi_1-\xi_2)} \quad (2.29)$$

$$I_1=\int_0^t e^{\gamma_1 t}e^{-\gamma_1 u}\sin\eta_1 u\,du \quad (2.30)$$

$$I_2=\int_0^t e^{\gamma_2 t}e^{-\gamma_2 u}\sin\eta_2 u\,du \quad (2.31)$$

$n_1=\lambda-\Phi$ (2.32)

$n_2=\lambda+\Phi$ (2.33)

Evaluating equations (2.30) and (2.31) using integration by part, one obtains

$$I_1=\frac{\xi_1}{(\eta_1^2+\xi_1^2)}\left(\frac{\eta_1}{\xi_1}\left[1-e^{-\xi_1 t}\cos\eta_1 t\right]-e^{-\xi_1 t}\sin\eta_1 t\right) \quad (2.34)$$

$$I_2=\frac{\xi_2}{(\eta_2^2+\xi_2^2)}\left(\frac{\eta_2}{\xi_2}\left[1-e^{-\xi_2 t}\cos\eta_2 t\right]-e^{-\xi_2 t}\sin\eta_2 t\right) \quad (2.35)$$

Substituting equations (2.34) and (2.35) into equation (2.28) gives

$$P_j(t)=\frac{P_c e^{\xi_1 t}\xi_1}{(\eta_1^2+\xi_1^2)}\left(\frac{\eta_1}{\xi_1}\left[1-e^{-\xi_1 t}\cos\eta_1 t\right]-e^{-\xi_1 t}\sin\eta_1 t\right)$$
$$-\frac{P_c e^{\xi_2 t}\xi_2}{(\eta_2^2+\xi_2^2)}\left(\frac{\eta_2}{\xi_2}\left[1-e^{-\xi_2 t}\cos\eta_2 t\right]-e^{-\xi_2 t}\sin\eta_2 t\right) \quad (2.36)$$

Subjecting equation (2.36) to further simplifications and rearrangement gives

$$P_j(t)=\frac{P_c\xi_1}{(\eta_1^2+\xi_1^2)}\left(\frac{\eta_1}{\xi_1}\left[e^{\xi_1 t}-\cos\eta_1 t\right]-\sin\eta_1 t\right)$$
$$-\frac{P_c\xi_2}{(\eta_2^2+\xi_2^2)}\left(\frac{\eta_2}{\xi_2}\left[e^{\xi_2 t}-\cos\eta_2 t\right]-\sin\eta_2 t\right) \quad (2.37)$$

Thus, in view of equation (2.7)

$$\varphi(x,t)=\sum_{j=1}^{n}\left\{\frac{P_c\xi_1}{(\eta_1^2+\xi_1^2)}\left(\frac{\eta_1}{\xi_1}\left[e^{\xi_1 t}-\cos\eta_1 t\right]-\sin\eta_1 t\right)\right.$$
$$\left.-\frac{P_c\xi_2}{(\eta_2^2+\xi_2^2)}\left(\frac{\eta_2}{\xi_2}\left[e^{\xi_2 t}-\cos\eta_2 t\right]-\sin\eta_2 t\right)\right\}\sin\frac{m\pi x}{L} \quad (2.38)$$

Equation (2.38) is the solution of the Rayleigh beam model resting on Filonenko-Borodich foundation.

Analysis for Resonance in the Dynamical System

In this dynamical system, the response amplitude of the system may grow without bound; this phenomenon is called resonance condition. Equation (2.38) clearly shows that the simply supported Rayleigh beam traversed by a moving load reached a state of resonance whenever

$$\xi_1=\xi_2, \qquad \eta_1^2=-\xi_1^2 \quad \text{or} \quad \eta_2^2=-\xi_2^2 \quad (2.39)$$

In this section, numerical results for the simply supported boundary condition are presented in plotted curves. An elastic beam of length 192.12m is considered Oni (1996). The mass is assumed to travel at initial velocity./128.m/s. Furthermore E, I π, μ are chosen to be 2.10924 x 10^9 N/m^2, 2.87698 x 10^{-3} m^4, 3.142857 and 2758.291kg/m respectively. The values of the tensile force T varied between 10 N and 2000 N, foundation stiffness K varied between 400 N/m^3 and 40000 N/m^3 rotatory inertia 0rvaried between 10and 40 the values of damping coefficient 0c is varied between 2.0 and 8.0. The results are as shown on the various graphs below for the simply supported boundary condition so far considered.

Fig.1 displays the displacement response of the simply supported Rayleigh beam to moving load for various values of tensile force. The graph showed that the response amplitude decreases as the value of Tincreases. In Fig. 2, the displacement response of the simply supported Rayleigh beam to moving load for various values of foundation stiffness is presented. The graph showed that the response amplitude decreases as the value of kincreases. Fig. 3 depicts the deflection profile of the simply supported Rayleigh beam to moving load for various values of rotatory inertia. It was observed that increase in the value of 0rresulted to decrease in the amplitude of vibration. In Fig. 4, the deflection profile of the simply supported Rayleigh beam to moving load for various values of damping coefficients has been presented. Also from the graph, increase in the values of 0c decreases the response amplitude of the beam. The comparison of the amplitude of vibration of Rayleigh beam on Winkler foundation and Filonenko-Borodich foundation has been depicted in Fig. 5. The result showed that the response amplitude of Winkler foundation is higher than that of F-B foundation.

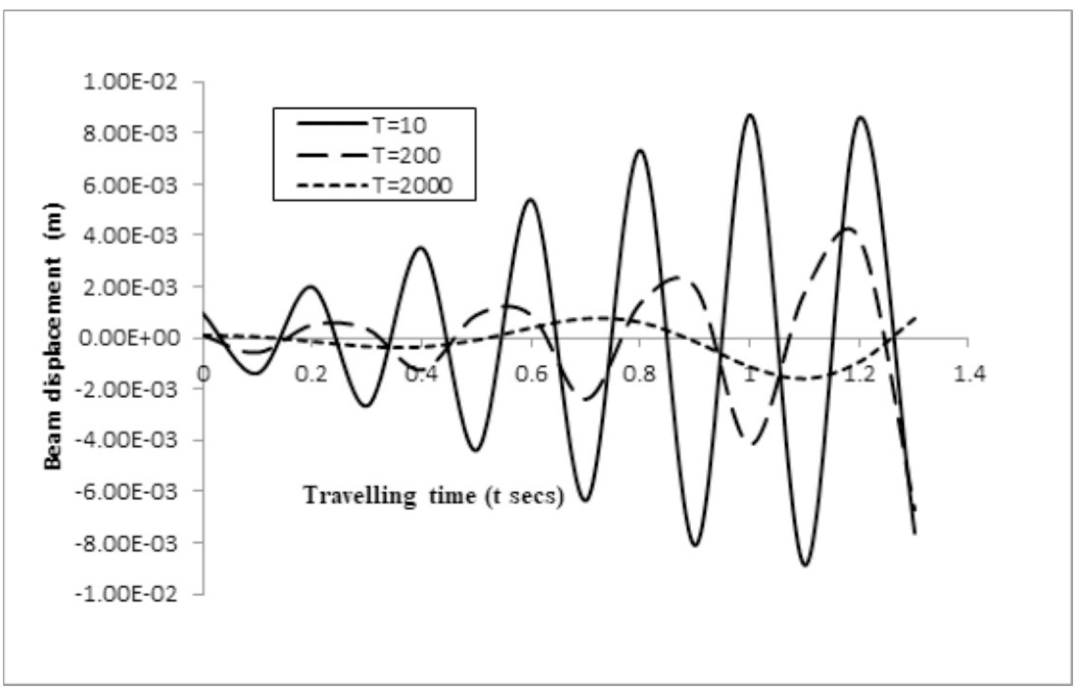

Figure 1: Response of a simply supported Rayleigh beam to moving loads for various values of Tensile force T.

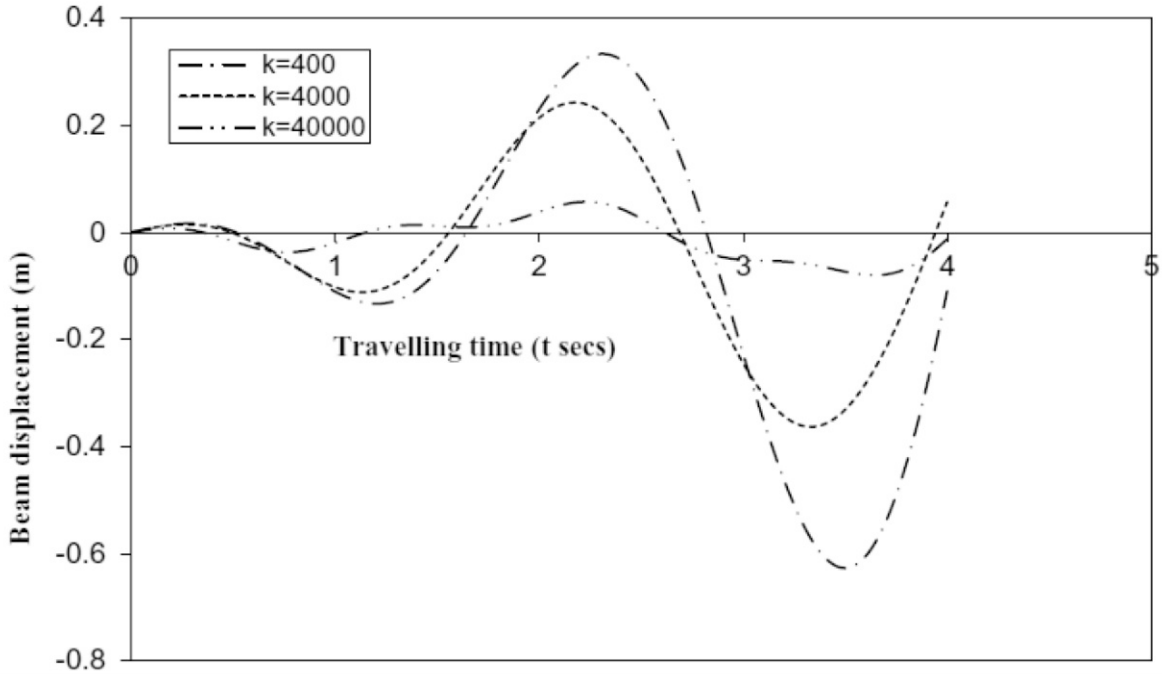

Figure 2: Response of a simply supported Rayleigh beam to moving loads for various values of foundation stiffness k.

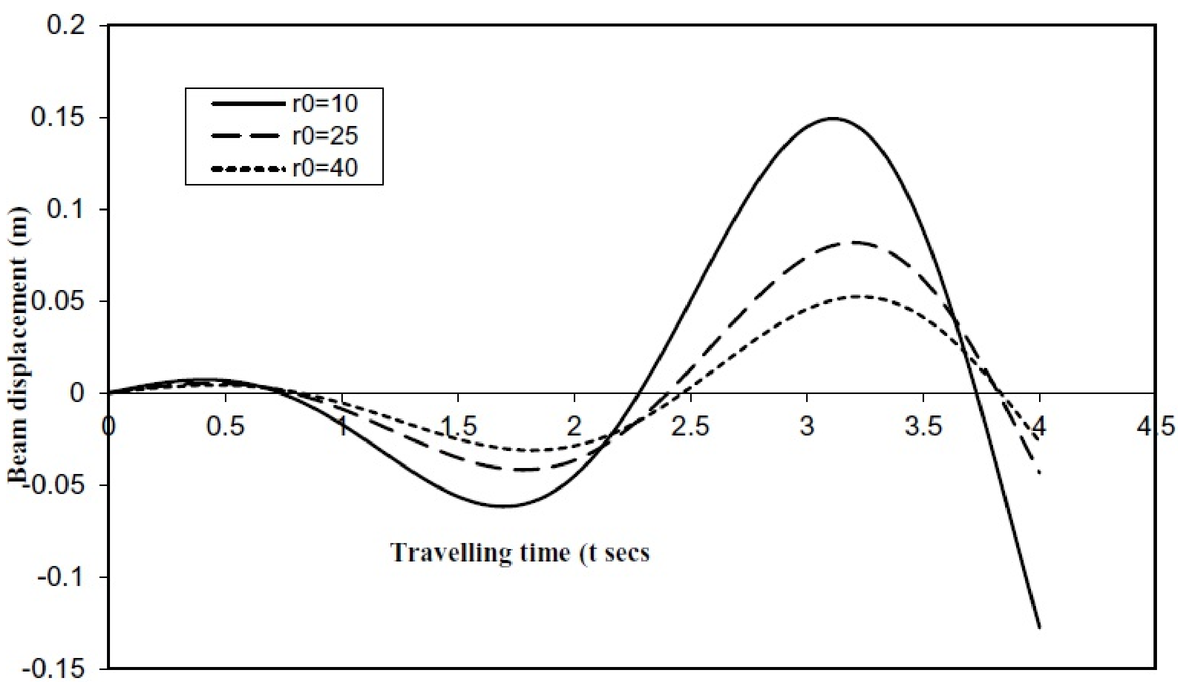

Figure 3: Response of a simply supported Rayleigh beam to moving loads for various values of Rotatory inertia r^0.

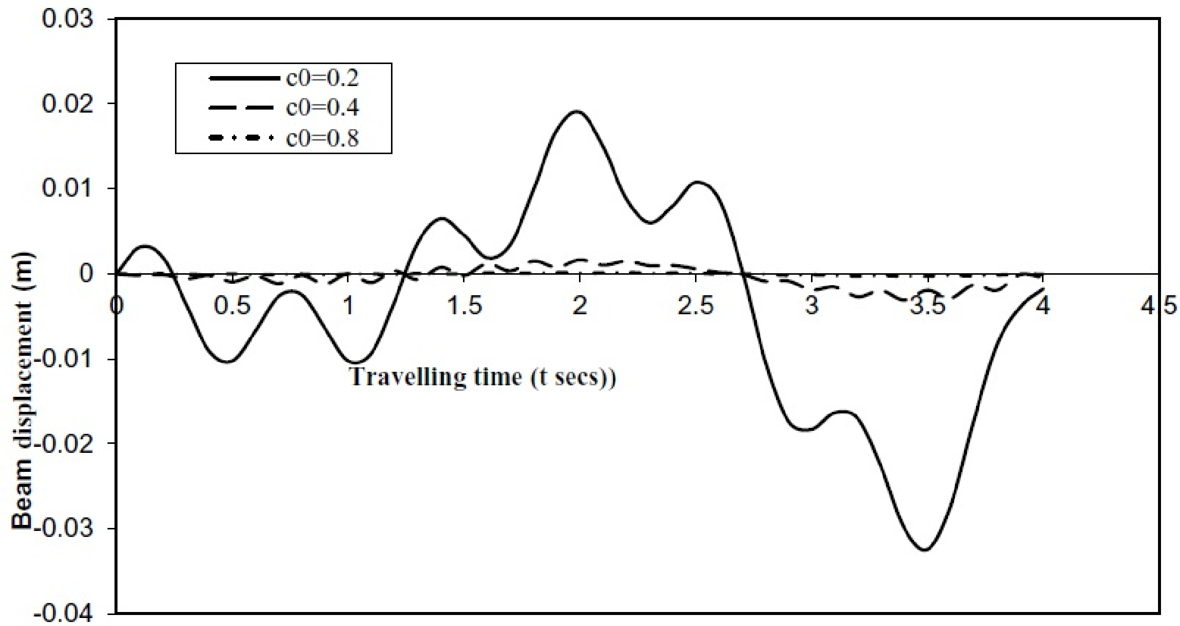

Figure 4: Response of a simply supported Rayleigh beam to moving loads for various values of damping coefficients c^0.

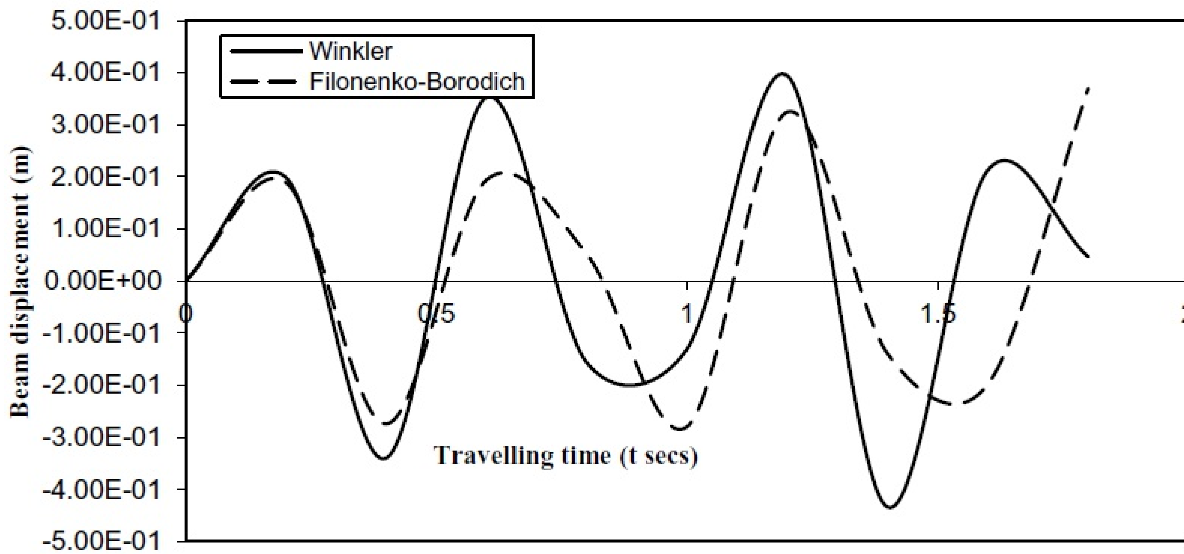

Figure 5: Comparison of the amplitude of vibration of a simply supported Rayleigh beam on Winkler model and Filonenko-Borodich Model.

Conclusion

The simplified approach by Filonenko-Borodich viewpoint for dynamic analysis of beams on foundation has been presented. To demonstrate the versatility of the Filonenko-Borodich foundation model, the results of the deflection for a simple beam resting on F-B foundation model are compared with the solution obtained by Winkler foundation model.

As a general observation, the obtained F-B solutions are more realistic and reasonable than those from Winkler foundation. It was also observed that increase in the values of some structural parameters namely, tensile force ,T foundation stiffnessk, rotatory inertia 0rand damping coefficients 0creduce the amplitude of the vibrating system involving beam under the action of moving loads, and hence the risk of resonance is drastically minimized.

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