



MAGNETIC FIELD EFFECTS ON REACTIVE FLOW IN ROTATING CONCENTRIC CYLINDER

¹Badejo, O. M and ²Usman, M. A

¹Department of Mathematical Sciences, Ondo State University of Science and Technology, Okitipupa.

²Department of Mathematical Sciences, Olabisi Onabanjo University, Ago Iwoye.

*Corresponding Author's e-mail: badeomi@yahoo.com

Abstract

The uneven road and overloading of moving vehicles and machines seems to be the major problems facing the reactive flow in rotating concentric cylinder in moving vehicles and machines. That is why this work investigates unsteady reactive flow of rotating concentric cylinder with reactive term having both cylinders rotating in opposite directions. The cylinders proximity is fixed to be 0.50mm, the magnetic field was investigated on other parameters in the governing equations. The governing equations were derived and Hartmann number (Ha) and some other parameters, were investigated guided by related journal's results. The governing equation were used to investigate the impacts of magnetic force on porosity, thermal conductivity, heat dissipation potential, velocity, temperature and the flow rate of the fluids in the rotating concentric cylinder (annulus). The energy equations were formulated with reactive term for proper results justification. Finite difference method were used to generate the solutions of both momentum and energy equations. MAPLE 18 software were used for the interpretation of figures and results. The velocity profile for values of Darcy number (Da) from $0.00mms^{-1}$ to $2.00mms^{-1}$ when the Ha is zero (0) it shows that the velocity is not smooth and as the Ha increases; velocity also increases. When all the parameters are visible then the Ha becomes curve which indicated proper fluid flow. The higher the Ha the more the fluid flow within the proximity. The temperature of all parameter increases as Ha increases. Excellent results were achieved during the research period.

Keywords: Reactive Term, Concentric Cylinder, Annulus, Magnetic Field and Finite Difference Method.

Introduction

The rate at which we feel changes in our body temperature as human being subject to ongoing body activity at a particular time is very important when we are talking about reactive flow in living things. Administering medication (drugs) to the body system (e.g. injection) and intake of water or food into human system are also one of the area considered when looking at reactive flow.

The reaction of fluid with the inner surface of an automobile engine can be seen as a related area to what the work is all about. Flow over rotating cylinders is important in a wide number of applications from shafts and axles to spinning projectiles. Also considered here is the flow in an annulus formed between the concentric cylinders where both of the cylindrical surfaces are rotating. As with the rotating flow associated with the cylinder, the

proximity of a surface is fixed.

Escudier *et al.*, (2002) evaluated the effect of internal cylinder rotation on the laminar flow of non-Newtonian fluids in an eccentric annular region, compared their numerical simulations of two-dimensional flow against experimental data. Their comparison of simulated velocity profiles and those obtained experimentally showed a good agreement. Okedayo *et al.* (2014) studied the effects of viscous dissipation on unsteady flow of a reactive fluid with a temperature dependent viscosity and thermal conductivity through a horizontal channel filled with porous material. The coupled nonlinear differential equations governing the flow were solved numerically using the semi-implicit finite difference scheme.

Ben and Henry (1996) investigated numerically the effect of a constant magnetic field on a three-dimensional buoyancy-induced flow in a cylindrical cavity, they put in light the structural changes of the flow induced by the magnetic field for each field orientation. Escudier *et al.* (2000) evaluated the effect of inner cylinder rotation on the laminar flow of Newtonian fluids in an eccentric annular region and described the effect of the friction factor under the influence of flow conditions and eccentricity. Ali (2014) estimated concentric annular flows of Newtonian fluids in vertical and horizontal arrangements based on Computational Fluid Dynamics (CFD) simulations, but did not quantify the effects of internal shaft rotation.

The study of flow and heat transfer characteristics in the vertical porous systems received attention by Kiwan and Alzahrani (2008) and Barletta *et al.*, (2008). Also, an investigation to provide an inverse approach for estimating the viscosity of the

fluid and thermal behavior of concentric cylinders has been presented by Hsu (2008). Kefeng *et al.* (2006) simulated numerically the characteristics of transient double-diffusive convection in a vertical cylinder using a finite element method. The Aim of this work is to study magnetic field effects on reactive flow in rotating concentric cylinder by investigating the unsteady flow in a concentric cylinder with and without reactive term considering the magnetic field on pressure gradient, energy dissipation, porosity, suction parameter and other reactive components of the system. Perturbation method was used to linearize the equations. MAPLE 14 and 18 software were used for the interpretations of results.

Mathematical Formulation

This section considered a system with reactive term flow in between concentric cylinders rotating simultaneously, unsteady state, laminar, and fully developed flow of fluids for which the density and the viscosity are constant.

The governing Navier-Stoke equations are:

Continuity equation:

$$\frac{\partial}{\partial r}(ur) = 0 \quad 1$$

$$\frac{\partial u}{\partial t} = v \left[\frac{\partial^2 u}{\partial r^2} + \frac{\partial}{\partial r} \left(\frac{u}{r} \right) \right] - \frac{1}{\rho} \frac{\partial P}{\partial r} - \frac{\sigma u B_0^2}{\rho} - \frac{vu}{K} - \frac{\Gamma u^2}{K} \quad 2$$

Energy equation:

$$\frac{\partial T}{\partial t} = \frac{u}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) K + \mu \left(\frac{\partial u}{\partial r} - \frac{u}{r} \right)^2 + QC_0 A e^{\left(\frac{\epsilon}{RT} \right)} \quad 3$$

Where u is the velocity, V is the fluid kinematic viscosity, r is the radius, $\frac{u}{r}$ is the rotating parameter in the governing partial differential equations.

Equations (3) is the energy equations with reactive term. The last term in equation (3) is the reactive term.

The Boundary Conditions are:

$$r = r_1, u = r_1\omega, r = r_2 \text{ and } u = r_2\omega \quad 4$$

Dimensionless Form:

$$U = \frac{u}{\omega b}, R = \frac{r}{b}, T = \frac{t}{t_0}, \theta = \frac{T}{T_0}, V_0 = \frac{\omega}{b} \text{ and } V = \frac{\mu}{\rho} \quad 5$$

These can also be written in the forms
 $u = U\omega b, r = Rb, t = Tt_0, T = \theta T_0,$
 $\omega = V_0\omega, \mu = V\rho$ 6

Substituting equation (6) into equations (2) yields

$$\frac{\partial U}{\partial T} = \frac{\partial^2 U}{\partial R^2} + \frac{1}{Re} \frac{\partial}{\partial R} \left(\frac{U}{R} \right) + \frac{G}{Re} - \frac{Ha^2}{Re} U - \frac{1}{Da} U - \frac{Fs Re}{Da} U^2 \quad 7$$

Substituting equation (6) into equations (3) yields

$$\frac{\partial \theta}{\partial T} = \frac{V_0}{Pr} \left(\frac{U}{R} \right) \frac{\partial}{\partial R} \left(R \frac{\partial \theta}{\partial R} \right) + Ec \left[\left(\frac{\partial U}{\partial R} \right)^2 - 2 \frac{U}{R} \left(\frac{\partial U}{\partial R} \right) + \left(\frac{U}{R} \right)^2 \right] + \gamma e^{\left(\frac{U}{R} \right)} \quad 8$$

Where

F_s: Forchheimer Geometric Inertial drag parameter

$$F_s = \frac{\Gamma}{b}$$

V₀: Suction Parameter

$$V_0 = \frac{\omega_2}{b}$$

G: Pressure Gradient

$$G = - \frac{\partial P}{\partial R}$$

γ: Frank-Kamenetskii Parameter

$$\gamma = \frac{QC_0 t_0}{T_0 b^2}$$

Re: Reynolds Number

$$Re = \frac{\omega_2 b^2}{\nu}$$

Ha: Hartmann Number

$$Ha^2 = \frac{\sigma B_0^2 b^2}{\mu}$$

Da: Darcy Number

$$Da = \frac{r}{b^2}$$

Pr: Prandtl Number

$$Pr = \frac{b^2}{Kt_0}$$

Ec: Eckert Number

$$Ec = \frac{\mu \omega_2^2 t_0}{T_0 b^2}$$

ε: Activation Energy

$$\epsilon = \frac{RT_0}{E}$$

B₀: Magnetic Field, **σ**: Fluid Electrical Conductivity, **ν**: Fluid Kinematic Viscosity **ρ**: Fluid Density, **k**: Porus Medium Permeability, **Q**: Heat Reaction, **A**: Rate Constant **R**: Universal Gas Constant, **C₀**: Initial Concentrating of the Reacting Species

The Boundary Conditions also yield

$$R = \frac{r_1}{b} \quad U = \frac{r_1 \omega}{\omega_2 b} \quad 9$$

Results and Discussion

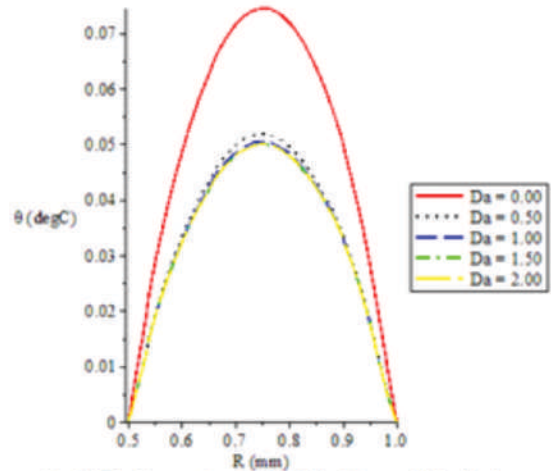


Fig. 1A The Temperature profile θ (deg C) versus Cylindrical radius R (mm) for values of Darcy number (Da) when $Ec=1, R=1, G=1, Ha=0, F_s=0.5, V_0=1$ & $Pr=1$

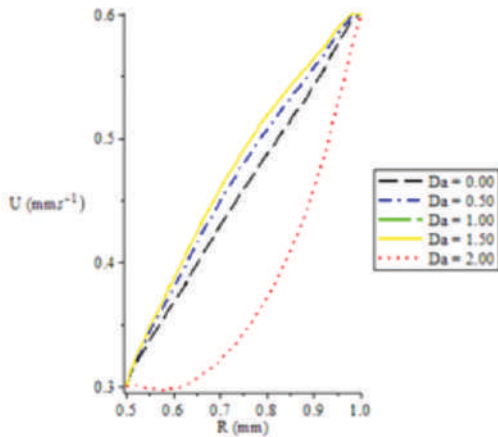


Fig. 1B The Velocity profile U (mm s^{-1}) versus Cylindrical radius R (mm) for values of Darcy number (Da) when $R=1, G=1, Ha=0, Fz=0.5$ & $V_0=1$.

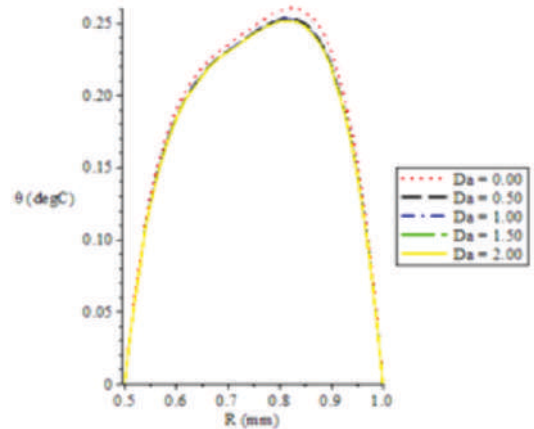


Fig. 3A The Temperature profile θ (deg C) versus Cylindrical radius R (mm) for values of Darcy number (Da) when $Ec=1, R=1, G=1, Ha=10, Fz=0.5, V_0=1$ & $Pr=1$

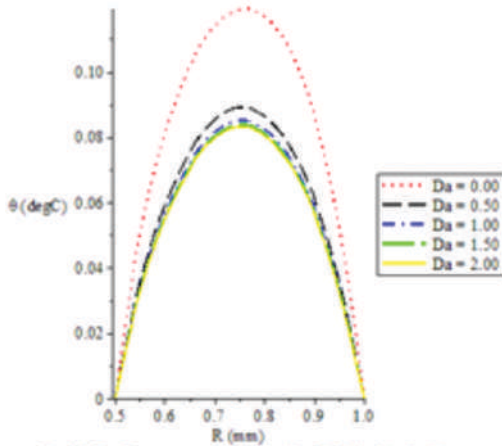


Fig. 2A The Temperature profile θ (deg C) Cylindrical radius R (mm) for values of Darcy number (Da) when $Ec=1, R=1, G=1, Ha=4, Fz=0.5, V_0=1$ & $Pr=1$

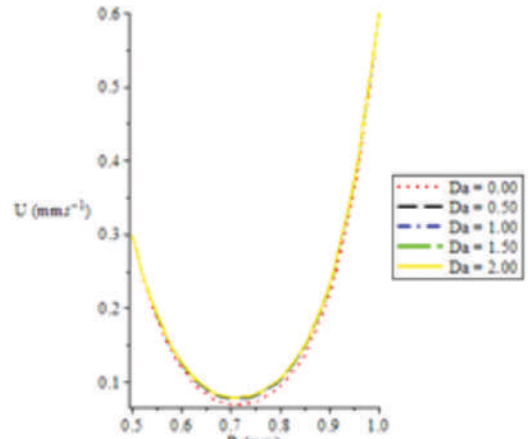


Fig. 3B The Velocity profile U (mm s^{-1}) versus Cylindrical radius R (mm) for values of Darcy number (Da) when $R=1, G=1, Ha=10, Fz=0.5$ & $V_0=1$.

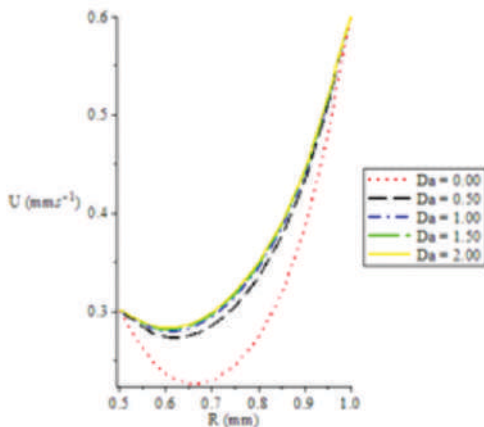


Fig. 2B The Velocity profile U (mm s^{-1}) versus Cylindrical radius R (mm) for values of Darcy number (Da) when $R=1, G=1, Ha=4, Fz=0.5$ & $V_0=1$.

Fig. 1A to Fig. 3A presents the temperature profile for different values of Hartmann number (Ha) on Darcy number (Da), as the Hartmann number increased from 0.00 to 10.00 the temperature profiles of the system reduced, the temperature increased as the values of dancy number (Da) increased. The temperature of the system is not stable when the Hartmann number (Ha) is less than 3.40 but it become more stable and closer as the value of the Hartmann number (Ha) increased.

Fig. 1B to Fig. 3B shows the velocity profiles of Hartmann number.

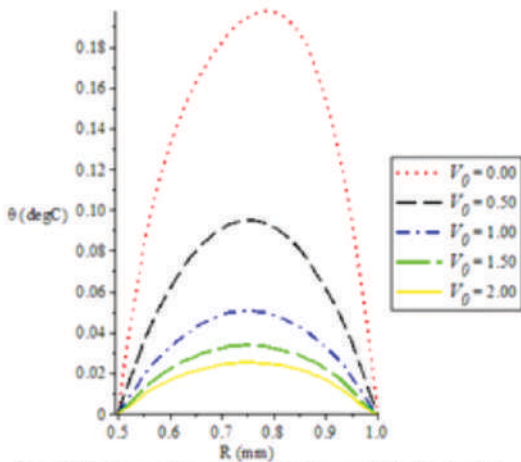


Fig. 4A The Temperature profile θ (deg C) versus Cylindrical radius R (mm) for values of Suction Parameter (V_0) when $Ha=0, R=1, Da=1, G=1, F_2=0.5, Ec=1$ & $Pr=1$

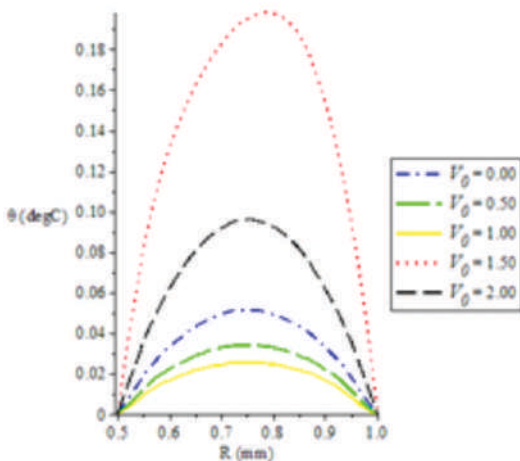


Fig. 4B The Temperature profile θ (deg C) versus Cylindrical radius R (mm) for values of Suction Parameter (V_0) when $Ha=1, R=1, Da=1, G=1, F_2=0.5, Ec=1$ & $Pr=1$

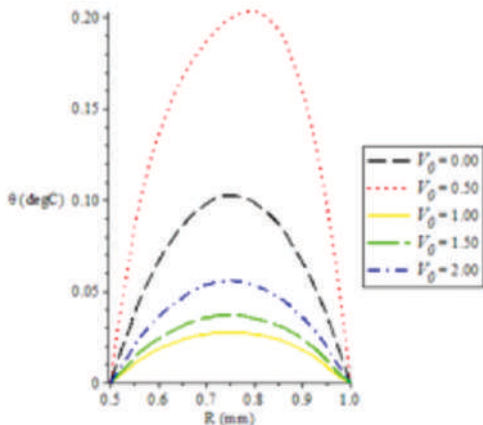


Fig. 4C The Temperature profile θ (deg C) versus Cylindrical radius R (mm) for values of Suction Parameter (V_0) when $Ha=2, R=1, Da=1, G=1, F_2=0.5, Ec=1$ & $Pr=1$

Fig. 4A to Fig. 4C shows the temperature profile of Hartmann number (Ha) values from 0.00 to 2.00 on different values of Suction parameter of a reactive system from 0.00 to 2.00. Fig. 4A with Hartmann number 0.00 shows that the relationship between Suction parameter and temperature is negative. Therefore, as the suction parameter values increases, the temperature also decreases, the system is stable and the highest temperature for this figure is about 19.00. Fig. 4B to Fig. 4C with Hartmann number values from 1.00 to 2.00 also show that the relationship between suction parameter and temperature is negative therefore as the suction parameter values increases, the temperature decreases, the system is not stable, not consistent and the highest temperature for these figures is about 0.21.

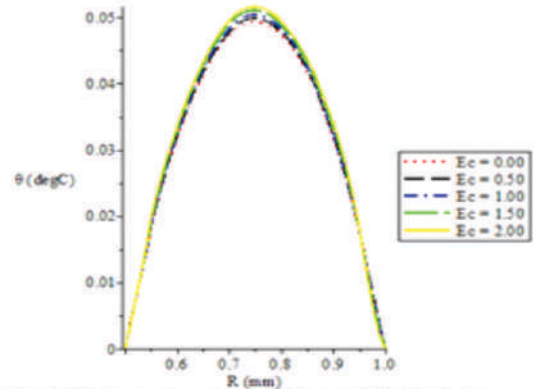


Fig. 5A The Temperature profile θ (deg C) versus Cylindrical radius R (mm) for values of Eckert (Ec) with reactive term, when $Ha=0, R=1, Da=1, G=1, F_2=0.5, V_0=1$ & $Pr=1$

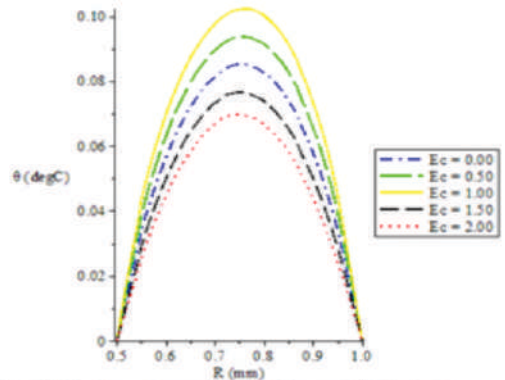


Fig. 5B The Temperature profile θ (deg C) versus Cylindrical radius R (mm) for values of Eckert (Ec) with reactive term, when $Ha=4, R=1, Da=1, G=1, F_2=0.5, V_0=1$ & $Pr=1$

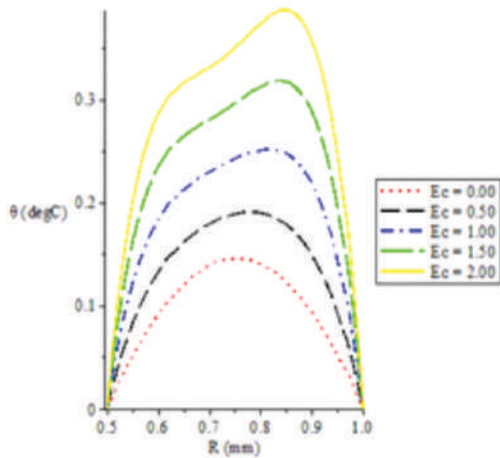


Fig. 5C The Temperature profile θ (deg C) versus Cylindrical radius R (mm) for values of Eckert (Ec) with reactive term, when $Ha=10, R=1, Da=1, G=1, Fz=0.5, V_0=1$ & $Pr=1$

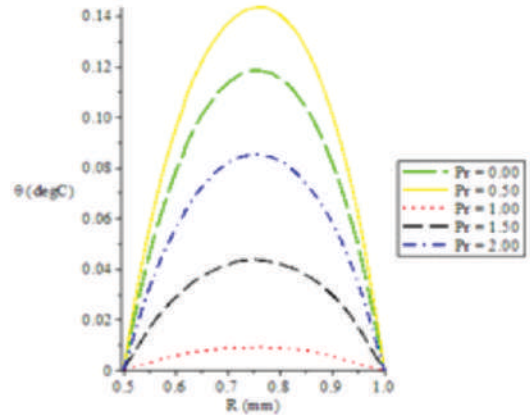


Fig. 6B The Temperature profile θ (deg C) versus Cylindrical radius R (mm) for values of Prandtl Number (Pr) when $Ec=1, R=1, Da=1, G=1, Fz=0.5, V_0=1$ and $Ha=4$

Fig. 5A to Fig. 5C shows the temperature profile of different values of Hartmann number (Ha) on values of Eckert number (Ec), as the Eckert number increased the temperature also increased likewise Hartmann number has positive effect on the system as it increased in values from 0.00 to 10.00. The temperature also increased and also allow free flow reaction in the reactive system. The system becomes more consistent and stable as the Hartmann number increased from 0.00 to 10.00.

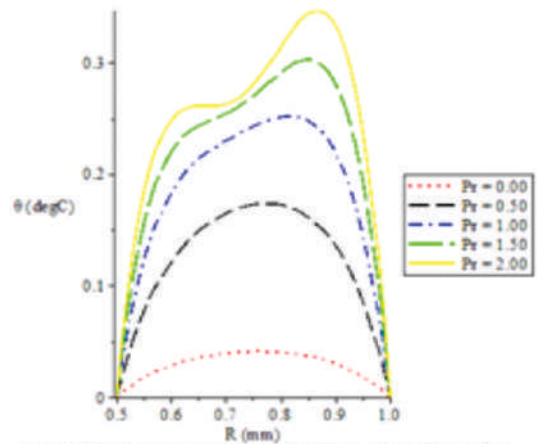


Fig. 6C The Temperature profile θ (deg C) versus Cylindrical radius R (mm) for values of Prandtl Number (Pr) when $Ec=1, R=1, Da=1, G=1, Fz=0.5, V_0=1$ and $Ha=10$

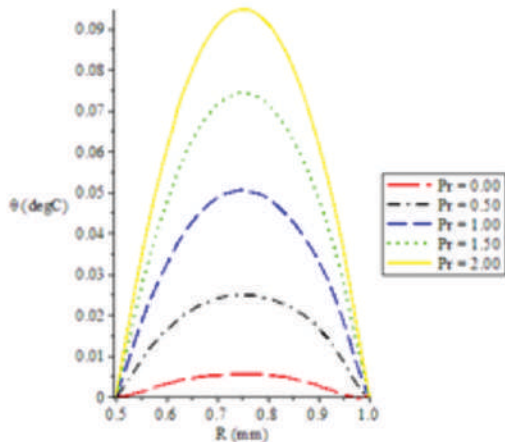


Fig. 6A The Temperature profile θ (deg C) versus Cylindrical radius R (mm) for values of Prandtl Number (Pr) when $Ec=1, R=1, Da=1, G=1, Fz=0.5, V_0=1$ and $Ha=0$

Fig. 6A to Fig. 6C shows the temperature profile of different values of Hartmann number (Ha) on values of Prandtl number (Pr), as the Prandtl number increased, the temperature also increased, likewise Hartmann number has positive effect on the system as it increased in values from 0.00 to 10.00 the temperature also increased and also allow free flow reaction in the reactive system. The system becomes more consistent and stable as the Hartmann number increased from 0.00 to 10,00 and the highest temperature of Fig. 6A to Fig. 6C increased to 0.09, 0.14 and 0.30 respectively.

Conclusion

The system with magnetic force will not work under a perfect condition, that is moving always on the road that is not even or experiencing external forces like over loading the system. Unsteady flow shown that time can determine the function of each parameter and it can be used to adjust and set time for temperature and the rate of fluid flow in the rotating concentric cylinder. Therefore, if there is humming or whirring noise while you are driving or the Automatic Brake System (ABS) light turns on, it may be time to change the vehicle's bearings.

It was observed that magnetic forces increased the temperature fluids flow in concentric cylinder. Over loading and movement of systems that are using rotating concentric cylinder (annulus) on uneven roads or surfaces increase it's temperature. Finally, when the Hartmann number increased the temperature increased and this resulted into gradual reduction of fluid in the reactive part of the rotating reactive cylinder. This usually leads to unwanted friction which can cause serious damage to the system. Applications of this study include magnetic material processing, mechanical engineering, wheel's hub or rotor and brake drum.

This study had shown that reactive flow in a rotating concentric cylinder (annulus) is very important in our day to day activities especially when we consider its benefits in all aspect to human being and the environment at large. Considering the results in the previous chapter, Hartmann number (Ha) which reveal the effect of magnetic force in this work with other parameters, shows that as the Hartmann number (Ha) increases on the other parameters the temperature of the system also increases which also increases the rate

of flow of the fluid in the proximity of the system.

Also, as the Hartmann number (Ha) increased, the temperature of Dancy number (Da) and other tested parameters also increased. Considering the components of the reactive term, it showed that all flowing fluids have the composition of the reactive term. The reactive term will only be sensed or obseved when there is increase in temperature as a result of external forces which increased the rate of Hartmann number (Ha) for Magnetic field and reduce the pressure gradient (G) for parameters like, Dancy number (Da) for porous median, Prandtl number (Pr) for heat convection and Eckert number (Ec) for dissipation of energy.

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