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UTILIZING THE POISSON PROBABILITY DISTRIBUTION SERIES FOR A SPECIFIC CLASS OF ANALYTIC FUNCTIONS THROUGH THE SALAGEAN DERIVATIVE OPERATOR AND STIRLING NUMBERS

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Abstract
In this investigation, we will utilize the Salagean derivative operator in conjunction with Stirling numbers to analyze the class T_v^t (Ω(s),θ) of analytic functions characterized by negative coefficients. We will establish both necessary and sufficient conditions, as well as inclusion relations, for series corresponding to the Poisson distribution. Furthermore, we will introduce an integral operator related to the Poisson distribution series, demonstrating its membership within this framework. A practical example will be provided to illustrate and underscore the connection between geometric function theory and statistics.
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Introduction

We begin by letting *H* denote the class of functions of analytic functions in the open disk $\{z :| z |< 1\}$ which has the following representation

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \qquad (1)$$

that are normalized by the condition $f(0) = 0$ and $f'(0) = 1$. Denote by *A* the subclasses of *H*, which consist of univalent function of the form

$$f(z) = z - \sum_{k=2}^{\infty} |a_k| z^k \qquad (2)$$

The well-known subclasses of *A* are starlike and convex functions of order zero are respectively denoted by S^* and *K*. Of particular interest to present researchers is the class of close-to-convex functions with the geometrical representation

$$Re\left(\frac{e^{i\delta} f'(z)}{g(z)}\right) > 0, \quad z \in D.$$

The class of all close-to-convex functions denoted by *C*. The first person to introduce the above function is Kaplan's (1952). If we let $g(z) \equiv z$ the class *C* reduces to class introduced by Mahzoon and Kargar's (2019) and is defined by

$$R = \left\{ f \in A : \left(e^{i\beta} f'(z)\right) > 0, \ \ for\ \beta \in R,\ z \in D \right\}$$

where *g* is convex function and δ ϵ R.

A function f of the form (2) belongs to class T_v^t ($\Omega(s),\theta$) if it satisfies the condition

$$Re = \left\{ \left(D^t f(z)\right)' + \frac{\Omega(s) + e^{i\theta}}{2} z\left(D^t f(z)\right)'' \right\} > v,$$

$$0 \le v < 1, \quad -\pi < \theta \le \pi \quad (3)$$

where $\Omega(s) = \frac{2}{1+e^{-s}}$ is the modified sigmoid function exploited in (Fadipe-Joseph et al., 2013) and s ≥ 0 (s is real), t = {0, 1, 2, 3, ...} and D_t is well-known Salangean differential operator.

Remark:1 If we let t = 0 and s = 0 in the definition above we obtain the class introduced and investigated by Mahzoon and Kargar (2019).

Remark 2: If s= 0 in definition 1 above we result to class explored and investigated in Awolere and Oladipo (2023)

In numerous applications involving physics, engineering, applied mathematics, and statistics, special functions are exploited. Number theory and special polynomials are closely related, and one of the most significant classes of special numbers is the Stirling numbers, which were established by the Scottish mathematician James Stirling in 1730 and they were rediscovered and given combinatorial significance in Rosen (2018). Some of the works on certain other special polynomials like Chebyshev and Gegenbauer polynomials can also be found in Oyekan and Aderibole (2020), Oyekan et al. (2023a), Oyekan et al. (2023b), and Oyekan et al. (2023c), to mention but a few.

A Stirling number of the second type in combinatorics, also known as a Stirling partition number, is the number of ways to divide a set of n objects into k nonempty subsets, and it is symbolized by S (n,k). These numbers can be found in the disciplines of partition theory and combinatorics. Using the method described by Frasin (2022), we will demonstrate in this study that the operator D^t of the S˘alagean differential may be represented in terms of Stirling numbers.

If a function f(z) belong to class A, we define

$D^0 f(z) = f(z)$, (4)

$D^1 f(z) = zf(z)$, (5)

And generally,

$D^t f(z) = z\,(D^{t-1} f(z))'$, t ϵ N, (6)

The derivative operator D^t was introduced by Anwar and Ahmad (2014).

We note that

$$D^t f(z) = z\,(D^{t-1} f(z))'$$

$$= \Phi_{t,1} z f'(z) + \Phi_{t,3} z^3 f'''(z) + \ldots + \Phi_{(t,t)} z^t f^t(z) \quad (7)$$

$$= \sum_{i=1}^{t} \Phi_{t,i}\, ztf^i(z), \quad (t \in N),$$

where

$$\Phi_{t,i} = i\Phi_{t-1,i} + \Phi_{t-1,i-1}, \quad and\ \Phi_{t,i} = \Phi_{t,t} = 1 \quad (8)$$

for instance

(I) if t = 2, we obtain

$$D^t f(z) = z\left(D^1 f(z)\right)' = zf'(z) + z^2 f''(z) = \Phi_{2,1} zf'(z) + \Phi_{2,2} z^2 f''(z) \quad (9)$$

where

$$\Phi_{2,1} = \Phi_{(2,2)}. \quad (10)$$

(II) If t = 3, we have

$$D^3 f(z) = z\left(D^2 f(z)\right)' = zf'(z) + 3z^2 f''(z) + z^3 f'''(z)$$
$$= \Phi_{3,1} zf'(z) + \Phi_{3,2} z^2 f''(z) + \phi_{3,3} z^3 f'''(z), \quad (11)$$

where

$$\phi_{3,2} = 2\phi_{2,2} + b_{2,1} = 3, \qquad \phi_{3,2} = \phi_{3,3} = 1. \quad (12)$$

(III) if t = 4, we note that

$$D^4 f(z) = z(D^3 f(z))' = zf('(z) + 7z^2 f''(z) + 6z^3 f'''(z) + z^4 f_{iv}(z)$$
$$= \Phi_{4,1} zf'(z) + \Phi_{4,2} z^2 f''(z) + \Phi_{4,3} z^3 f'''(z) + \Phi_{4,3} + \Phi_{4,4} z^4 f^{iv}(z) \quad (13)$$

where

$$\Phi_{4,2} = 2b_{3,2} + \Phi_{3,1} = 7,\ \Phi_{4,3} = 3b_{3,3} + \Phi_{3,2} = 6,\ \Phi_{4,1} = b_{4,4} = 1. \quad (14)$$

We observe from Awolere and Oladipo (2023) that

(1) $\Phi_{t,i} = t\Phi_{t-1,i} + \Phi_{t-1,i-1}$,

(2)$\Phi_{t,i-1} = (t(t-1))/2)$,

(3)$\Phi_{t,2} = 2^{t-1} - 1$, (4)$\Phi_{t,3} = (1/6)(3^t - 3.2^t + 3)$, (5)$\Phi_{t,1} = \Phi_{t,t} = 1$,(6)$\Phi_{t,i}\ 0, when\ i > t$, (7)$\Phi_{p,i} \equiv 0 (mod p)\ if\ f\ 1 < t < p$,

where p is a prime number.

Moreover, for t = 2,3,4, we infer that

$D^2 f(z) = z^2 f''(z) + zf'(z)$

$$z+\sum_{k=2}^{\infty}k^2a_kz^k=z+\sum_{k=2}^{\infty}[k(k-1)+k]a_kz^k, \quad (15)$$

$$D^3f(z)=z^3f'''(z)+3z^2f''(z)+zf'(z),$$

$$z+\sum_{k=2}^{\infty}k^3a_kz^k=z+\sum_{k=2}^{\infty}[k(k-1)(k-2)+3k(k-1)+k]a_kz^k, \quad (16)$$

and

$$D^4f(z)=z^4f''''(z)+6z^3f'''(z)+7z^2f''(z)+zf'(z),$$

$$z+\sum_{k=2}^{\infty}k^4a_kz^k=z+\sum_{k=2}^{\infty}[k(k-1)(k-2)(k-3)$$

$$+6k(k-1)(k-2)+7k(k-1)+k]a_kz^k \quad (17)$$

we conclude from (15) - (17)

$$k=(k-1)+1=\Phi_{2,2}(k-1)+\Phi_{2,1},$$

$$k^2=(k-1)(k-2)+3(k-1)+1=\Phi_{3,3}$$

$$(k-1)(k-2)+\Phi_{3,2}(k-1)+\Phi_{3,1} \quad (18)$$

$$k_3=(k-1)(k-2)(k-3)+$$

$$6(k-1)(k-2)+7(k-1)+1,$$

$$=\Phi_{4,4}(k-1)(k-2)(k-3)+\Phi_{4,3}(k-1)$$

$$(k-2)+\Phi_{4,2}(k-1)+\Phi_{4,1}. \quad (19)$$

A variable Y is said to be Poisson distributed if it takes the values 0,1,2,3,4,... with the probabilities

$$e^{-m},\frac{me^{-m}}{1!},\frac{m^2e^{-m}}{2!},\frac{m^3e^{-m}}{3!}$$ ...respectively, where m is called the parameter. Thus

$$p(Y=r)=\frac{m^re^{-m}}{r!},\ r=0,1,2,3,4,.....$$

A power series whose coefficients are probabilities of Poisson distribution was introduced and studied recently by Porwal (2014) as

$$P_0(m,z)=z+\sum_{k=2}^{\infty}\frac{m^{k-1}e^{-m}}{(k-1)!}z^k, \quad z\in\Delta, \quad (20)$$

Where m > 0, which was further studied by Frazin (2020).

Equation (6) has numerous applications in the actual world, as described in the literature. It has applications in software defect control, the modeling of the distribution of overlapping word occurrences, the modeling of DNA substitution, and traffic accident data as discussed by Anwar and Ahmad, (2014). Inspired in large part by recent work by Awolere and Oladipo (2023) and that of earlier works by Frasin (2022, 2019), Murugusundaramoorthy et al. (2016), Porwal (2020a, 2018), and Porwal and Ahmad (2020b), we determine necessary and sufficient criteria for Poisson distribution class Γ (m,z) to be in the following derived classes T_v^0(θ, Ω,(s)), T_v^1 (θ, Ω,(s)), and T_v^2 (θ, Ω,(s)). In what follows, we state conditions for the integral

$$\Gamma(m,z)=\int_0^z\epsilon^{-1}K(m)(\epsilon)d\epsilon$$

to be in the class T_v^0(θ, Ω,(s)), T_v^1(θ, Ω,(s)), and T_v^2 (θ, Ω,(s))

The following Lemmas shall be recalled for our investigation.

Lemma 1: A function f of the form (2) belong to T_v^t (θ, Ω,(s)) if and only if

$$\sum_{k=2}^{\infty}k^{t+1}[2+(k-1)(\Omega(s)+\cos\theta)]|ak|\le 2(1-v) \quad (21)$$

The result (6) is sharp for the relation

$$f(z)=z-\frac{2(1-y)}{k^t[2k+k(k-1)(\Omega(s)+\cos\theta)]}z^k,\ \ k\le 2.$$

Proof: Suppose that the function T_v^t (θ, Ω,(s)). Then by (3) we note that

$$R\left\{1-\sum_{k=2}^{\infty}k^t\left[\frac{2k+k(k-1)(\Omega(s)+\cos\theta)}{2}\right]|a_k|z^{k-1}\right\}>v$$

Choose z to be real and let $z\to 1^-$ we obtain

$$1-\sum_{k=2}^{\infty}k^t\left[\frac{2k+k(k-1)(\Omega(s)+\cos\theta)}{2}\right]|a_k|>v$$

which is equivalent to (21). Conversely, suppose that (21) is true, then, we have

$$\left|\left(D^tf(z)\right)'+\frac{\Omega(s)+e^{i\theta}}{2}z\left(D^tf(z)\right)''\right|<$$

$$\sum_{k=2}^{\infty}k^t[2k+k(k-1)(\Omega(s)+\cos\theta)]|a_k|\le 2(1-v),$$

which implies that $f(z)\in T_v^t(\theta,\Omega(s))$.

Lemma 2. Dixit and Pal (1995). If f of the form (1) belongs to class $R^T(A,B)$ then

$$|a_k| \le \frac{(A,B)|T|}{k}, k \in N - \{1\}.$$

the result is sharp for

$$f(z) = \int_0^z \left[1 + (A+B)\frac{Tt^{t-1}}{1+Bt^{t-1}}\right] dt, \quad (z \in D, \ k \in N - \{1\}).$$

2 THE NECESSARY AND SUFFICIENT CONDITIONS

For easy handling throughout the sequel, we make use of the fact that

$$\sum_{k=2}^{\infty} \frac{m^{k-1}}{(k-1)!} = e^m - 1 \quad and$$

$$\sum_{k=j}^{\infty} \frac{m^{k-1}}{(k-1)!} = m^{j-1}e^m, \quad j \ge 2. \tag{22}$$

Theorem 1: Let m > 0, 0 ≤ v < 1 and s ≥ 0 (s is real), then Γ (m,z) є T_v^0(θ, Ω (s)) if and only if

$$\Phi_{3,3}[\Omega(s) + cos\theta]m^2 + [\Phi_{3,2}(\Omega(s) + cos\theta) + \Phi_{2,2}(2 - \Omega(s) - cos\theta)]m + [\Phi_{3,1}(\Omega(s) + cos\theta) + \Phi_{2,1}(2 - \Omega(s) - cos\theta)](1 - e^{-m}) \le 2(1 - v). \tag{23}$$

Proof. Since P_0(m,z) is defined by (20), by Lemma 1 it is suffice to establish that

$$\Delta_1 = \sum_{k=2}^{\infty} k[2 + (k-1)(\Omega(s) + \cos\theta)]\frac{m^{k-1}}{(k-1)!} \le 1(1-v)e^m$$

we set

$$k = \Phi_{2,2}(k-1) + \Phi_{2,1}, \ \ k2 = \Phi_{3,3}(k-2)(k-2) + \Phi_{3,2}(k-1) + \Phi_{3,1}.$$

Thus

$$\Delta_1 = \phi_{3,3}(\Omega(s) + cos\theta)\sum_{k=2}^{\infty}(k-1)(k-2)\frac{m^{k-1}}{(k-1)!} +$$

$$[\phi_{3,2}(\Omega(s) + cos\theta) + \phi_{2,2}(\Omega(s) - cos\theta\sum_{k=2}^{\infty}(k-1)\frac{m^{k-1}}{(k-1)!} +$$

$$[\phi_{3,1}(\Omega(s) + cos\theta) + \phi_{2,1}(\Omega(s) - cos\theta\sum_{k=2}^{\infty}(k-1)\frac{m^{k-1}}{(k-1)!} +$$

$$= \phi_{3,3}(\Omega(s) - cos\theta\sum_{k=2}^{\infty}\frac{m^{k-1}}{(k-1)!} +$$

$$[\phi_{3,2}(\Omega(s) + cos\theta) + \phi_{2,2}(\Omega(s) - cos\theta\sum_{k=2}^{\infty}\frac{m^{k-1}}{(k-2)!} +$$

$$[\phi_{3,1}(\Omega(s) + cos\theta) + \phi_{2,1}(\Omega(s) - cos\theta\sum_{k=2}^{\infty}\frac{m^{k-1}}{(k-1)!}.$$

We conclude by applying condition (22) on the equation above, we have

$$= \Phi_{3,3}[\Omega(s) + cos\theta]m^2 + [\Phi_{3,2}(\Omega(s) + cos\theta) + \Phi_{2,2}(\Omega(s) - cos\theta)]m + [\Phi_{3,1}(\Omega(s) + cos\theta) + \Phi_{2,1}(\Omega(s) - cos\theta)](1 - e^{-m}).$$

Hence, the last expression is bounded by 2 (1-v), if (23) holds and which complete the proof.

Theorem 2: Let m > 0, 0 ≤ v <1 and s ≥ 0 (s is real) then Γ (m,z) є T_v^1 (θ,Ω (s)) if and only if

$$\Phi_{4,4}[\Omega(s) + cos\theta]m^3 + [\Phi_{4,3}(\Omega(s) + cos\theta) + \Phi_{3,3}(2 - \Omega(s) - cos\theta)]m^2 [\Phi_{4,2}(\Omega(s) + cos\theta) + \Phi_{3,2}(\Omega(s) - cos\theta)]m + [\Phi_{4,1}(\Omega(s) + cos\theta) + \Phi_{3,1}(2 - \Omega(s) - cos\theta)](1 - e^{-m}) \le 2(1 - v). \tag{24}$$

Proof. From Lemma 1, we only need to prove that

$$\Delta_2 = \sum_{k=2}^{\infty} k^2[2 + (k-1)(\Omega(s) + cos\theta)]\frac{m^{k-1}}{(k-1)!} \le 1(1-v)e^m$$

$$= \sum_{k=2}^{\infty}[\Omega(s) + cos\theta)k^3 + (2 - \Omega(s) - cos\theta)k^2]\frac{m^{k-1}}{(k-1)!}$$

By (18) and (19) with simple computation, we get

$$\Delta_2 = \phi_{4,4}(\Omega(s) + cos\theta)\sum_{k=2}^{\infty}(k-1)(k-2)(k-3)\frac{m^{k-1}}{(k-1)!} +$$

$$[\phi_{4,3}(\Omega(s) + cos\theta) + \phi_{3,3}(2 - \Omega(s) - cos\theta]\sum_{k=2}^{\infty}(k-1)(k-2)\frac{m^{k-1}}{(k-1)!}$$

$$+[\phi_{4,2}(\Omega(s) + cos\theta) + \phi_{3,2}(2 - \Omega(s) - cos\theta\sum_{k=2}^{\infty}(k-1)\frac{m^{k-1}}{(k-1)!}$$

$$+[\phi_{4,1}(\Omega(s) + cos\theta) + \phi_{3,1}(2 - \Omega(s) - cos\theta\sum_{k=2}^{\infty}\frac{m^{k-1}}{(k-1)!}$$

$$= \phi_{4,4}(\Omega(s) - cos\theta\sum_{k=2}^{\infty}\frac{m^{k-1}}{(k-4)!} +$$

$$[\phi_{4,3}(\Omega(s) + cos\theta) + \phi_{3,3}(2 - \Omega(s) - cos\theta]\sum_{k=2}^{\infty}\frac{m^{k-1}}{(k-3)!}$$

$$+[\phi_{4,2}(\Omega(s) + cos\theta) + \phi_{3,2}(2 - \Omega(s) - cos\theta\sum_{k=2}^{\infty}\frac{m^{k-1}}{(k-2)!}$$

$$+[\phi_{4,1}(\Omega(s) + cos\theta) + \phi_{3,1}(2 - \Omega(s) - cos\theta\sum_{k=2}^{\infty}\frac{m^{k-1}}{(k-1)!}$$

and by (22) ,we observe

$$\Delta_2 = \Phi_{4,4}[\Omega(s) + cos\theta]m^3 + \big[\Phi_{4,3}(\Omega(s) + cos\theta) + \Phi_{3,3}(2 - \Omega(s) - cos\theta)\big]m^2 + [\Phi_{4,2}(\Omega(s) + cos\theta) + \Phi_{3,2}(\Omega(s) - cos\theta)]m + [\Phi_{4,1}(\Omega(s) + cos\theta) + \Phi_{3,1}(2 - \Omega(s) - cos\theta)](1 - e^{-m}).$$

The last expression is bounded above by 2 (1-v), if and only if (23) satisfied.

Theorem 3: Let m > 0, 0≤v<1 and s ≥ 0 (s is real), then Γ (m,z) ϵ T_v^2 (θ,Ω (s)) if and only if

$$\Phi_{5,5}[\Omega(s) + cos\theta]m^4 + \big[\Phi_{5,4}(\Omega(s) + cos\theta) + \Phi_{4,4}(2 - \Omega(s) - cos\theta)\big]m^3 + \big[\Phi_{5,3}(\Omega(s) + cos\theta) + \Phi_{4,3}(\Omega(s) - cos\theta)\big]m^2 + [\Phi_{5,2}(\Omega(s) + cos\theta) + \Phi_{4,2}(2 - \Omega(s) - cos\theta)]m + [\Phi_{5,1}(\Omega(s) + cos\theta) + \Phi_{4,1}(2 - \Omega(s) - cos\theta)](1 - e^{-m}) \leq 2(1 - \nu). \quad (25)$$

Proof: From Lemma 1, we only need to prove that

$$\Delta_3 = \sum_{k=2}^{\infty} k^3[2 + (k-1)(\Omega(s) + cos\theta)]\frac{m^{k-1}}{(k-1)!} \leq 1(1-\nu)e^m$$

$$= \sum_{k=2}^{\infty}[(\Omega(s) + cos\theta)k^4 + (2 - \Omega(s) - cos\theta)k^3]\frac{m^{k-1}}{(k-1)!}$$

by (19) and

$K^4 = (k-1)(k-2)(k-3)(k-4)+10(k-1)(k-2)(k-3)+6(k-1)(k-2)+7(k-1)+1, = \Phi_{5.5}(k-1)(k-2)(k-3)(k-4)+\Phi_{5.4}(k-1)(k-2)(k-3)+\Phi_{5.3}(k-1)(k-2)+\Phi_{5.2}(k-1)+\Phi_{5.1}$

and with simple computation in the above equation, we arrived at the desired result.

From Lemma 2, we will study the impact of Poisson distribution series on classes class T_v^0(θ, Ω,(s)), T_v^1 (θ, Ω,(s)), and T_v^2(θ, Ω,(s)).

Theorem 4: Let m > 0, 0 ≤ v < 1 and s ≥ 0 (s is real) and f ϵ RT (A,B) then Γ (m,z) T_v^0(θ,Ω (s)), T_v^1 (θ,Ω (s)) and T_v^2 (θ,Ω (s)), if and only if conditions

$$(A - B)|T|\Big(\Phi_{2,2}(\Omega(s) + cos\theta)m + \big[\Phi_{2,1}(\Omega(s) + cos\theta) + \Phi_{2,1}(2 - \Omega(s) - cos\theta)\big](1 - e^{-m})\Big) \leq 2(1 - \nu), \quad (26)$$

$$(A - B)|T|\Big(\Phi_{3,3}[\Omega(s) + cos\theta]m^2 + \big[\Phi_{3,2}(\Omega(s) + cos\theta) + \Phi_{2,2}(2 - \Omega(s) - cos\theta)\big]m + \big[\Phi_{3,1}(\Omega(s) + cos\theta) + \Phi_{2,1}(2 - \Omega(s) - cos\theta)\big](1 - e^{-m})\Big) \leq 2(1 - \nu)$$

and

$(A - B)|T|$ $(\Phi_{4.4}[\Omega(s)+ \cos\theta]m^3 + \Phi_{4.3}[\Omega(s)+ \cos\theta] + \Phi_{3.3}(2 - \Omega(s) - \cos\theta)]m^2 + [(\Phi_{4.2}(\Omega(s)+ \cos\theta) + (\Phi_{3.2}(\Omega(s) - \cos\theta]m + [\Phi_{4.1}[\Omega(s)+ \cos\theta] +$ & $(\Phi_{3.1}(2 - \Omega(s) - \cos\theta)](1 - e^{-m})) \leq 2(1 - v)$ (28)

are respectively satisfied

INTEGRAL TRANSFORM

In this section we obtain necessary and sufficient conditions for integral operator

$$\Gamma(m)(z) = \int_0^z \frac{\Gamma(m)(\varepsilon)}{\varepsilon}d\varepsilon$$

to be in the classes T_v^0 θ,Ω (s)), T_v^1(θ,Ω (s)) and T_v^2 (θ,Ω (s)),

Theorem 5: Let m > 0, 0 ≤ v < 1 and s ≥ 0 (s is real) and f ϵ RT (A,B) then Γ (m,z) T_v^0(θ,Ω (s)), T_v^1 (θ,Ω (s)) and T_v^2(θ,Ω (s)), if and only if conditions

$(\Phi_{2.2}(\Omega(s)+ \cos\theta]m + [\Phi_{2.1}(\Omega(s) + \cos\theta) + \Phi_{2.1}(2 - \Omega(s) - \cos\theta)] (1 - e^{-m})) \leq 2 (1 - v)$ (29)

$(\Phi_{3.3}[\Omega(s)+ \cos\theta]m^2 + [\Phi_{3.2}(\Omega(s) + \cos\theta) + \Phi_{2.2}(2 - \Omega(s) - \cos\theta)]m$& $+ \Phi_{3.1} (\Omega(s) + \cos\theta) + \Phi_{2.1}(2 - \Omega(s) - \cos\theta)] (1 - e^{-m})) \leq 2 (1 - v)$ (30)

and

$(\Phi_{4.4}[\Omega(s)+ \cos\theta]m^3 + \Phi_{4.3}[\Omega(s)+ \cos\theta] + \Phi_{3.3}(2 - \Omega(s) - \cos\theta)]m^2 + [(\Phi_{4.2}(\Omega(s)+ \cos\theta) + (\Phi_{3.2}(\Omega(s) - \cos\theta]m + [\Phi_{4.1}[\Omega(s)+ \cos\theta] +$ &$\Phi_{3.1}(2 - \Omega(s) - \cos\theta)](1 - e^{-m})) \leq 2 (1 - v)$ (31)

are respectively satisfied

Example for case study 1

In a certain factory producing cycle tyres, there is a small chance of 1 in 500 tyres to be defective. The tyres are supplied in lots of 10. Determine the approximate number of lots containing no defective in a consignment of 10000 lots.

Solution

$$\Pr obability = \frac{1}{500}, n = 10, m = np = 10x\frac{(1)}{500} = 0.02$$

$$\Pr obability \text{ of no defective} = p(0) = \frac{e^{(0.02)}(0.02)^0}{0!} = e^{-0.02}$$

$$= 0.9802$$

Corollary 1: Let m = 0.0020, $0 \le v < 1$ and $s \ge 0$ (s is real). Then Γ (m,z)T_v^0 (θ,Ω (s)),T_v^1 (θ,Ω (s)) and $T_v^2(\theta,\Omega (s))$, if and only if conditions

$\Phi_{3.3}[\Omega(s)+ \cos\theta]\, 4 \times 10^{-4} + [\Phi_{3.2}(\Omega(s)+ \cos\theta) + \Phi_{2.2}(2 - \Omega(s) - \cos\theta)]\, 2 \times 10^{-3} + [(\Phi_{3.1}(\Omega(s)+ \cos\theta) + \Phi_{2.1}(\Omega(s) - \cos\theta]\, 1.98 \times 10^{-2} \le 2\,(1-v)$,

$\Phi_{4.4}[\Omega(s)+ \cos\theta]\, 8 \times 10^{-6} + [\Phi_{4.3}(\Omega(s)+ \cos\theta) + \Phi_{3.3}(2 - \Omega(s) - \cos\theta)]\, 4 \times 10^{-4} + \Phi_{4.2}(\Omega(s)+ \cos\theta) + \Phi_{3.2}(2 - \Omega(s) - \cos\theta)]\, 2 \times 10^{-2} + (2 - \Omega(s) - \cos\theta)] + \Phi_{4.1}(\Omega(s)+ \cos\theta) + \Phi_{3.1}(2 - \Omega(s) - \cos\theta)]\, 1.98 \times 10^{-2} \le 2\,(1-v)$

and

$\Phi_{5.5}[\Omega(s)+ \cos\theta]\, 1.6 \times 10^{-7} + [\Phi_{5.4}(\Omega(s)+ \cos\theta) + \Phi_{4.4}(2 - \Omega(s) - \cos\theta)]\, 8 \times 10^{-6} + [\Phi_{5.3}(\Omega(s)+ \cos\theta) + \Phi_{4.3}(2 - \Omega(s) - \cos\theta)]\, 4 \times 10^{-4} + \Phi_{5.2}(\Omega(s)+ \cos\theta) + \Phi_{4.2}(2 - \Omega(s) - \cos\theta)]\, 0.02 + [\Phi_{5.1}(\Omega(s)+ \cos\theta) + \Phi_{4.1}(2 - \Omega(s) - \cos\theta)]\, 1.98 \times 10^{-2} \le 2\,(1-v)$

Corollary 2: Let m = 0.02, $0 \le v < 1$ and $s \ge 0$ (s is real) then Γ (m,z) $T_v^0(\theta,\Omega (s))$, $T_v^1(\theta,\Omega (s))$ and $T_v^2(\theta,\Omega (s))$, if and only if conditions

$\Phi_{3.3}[1+ \cos\theta]0.02^2 + [\Phi_{3.2}[1+ \cos\theta] + \Phi_{2.2}(1 - \cos\theta)]\, 0.02 + [\Phi_{3.1}(1+ \cos\theta) + \Phi_{2.1}(1 - \cos\theta)]\, 0.0198 \le 2\,(1-v)$

$\Phi_{4.4}(1 + \cos\theta)]\, 0.02^3 + [\Phi_{4.3}(1 + \cos\theta) + \Phi_{3.3}(1 - \cos\theta)]\, 0.02^2 n + [\Phi_{4.2}(1+ \cos\theta) + \Phi_{3.2}(2 - \Omega(s) - \cos\theta)]\, 0.02 + [\Phi_{4.1}(1+ \cos\theta) + \Phi_{3.1}(1 - \cos\theta)]\, 0.0198 \le 2\,(1-v)$

and

$\Phi_{5.5}[1+ \cos\theta]0.02^4 + [\Phi_{5.4}[1+ \cos\theta] + \Phi_{4.4}(1 - \cos\theta)]\, 0.02^3 + [\Phi_{5.3}(1+ \cos\theta) + \Phi_{4.3}(1 - \cos\theta)]\, 0.02^2$

$\Phi_{5.2}(\Omega(s) + \cos\theta)] + [\Phi_{4.2}(2 - \Omega(s) - \cos\theta)]\, 0.02 + [\Phi_{5.1}(\Omega(s) + \cos\theta) + \Phi_{4.1}(2 - \Omega(s) - \cos\theta)]\, 0.0198 \le 2\,(1-v)$

Corollary 3: Let m = 0.02, $0 \le v < 1$ and $s \ge 0$ (s is real). Then Γ (m,z)$T_0^0(\theta,\Omega (s))$, $T_0^1(\theta,\Omega (s))$ and $T_0^2(\theta,\Omega (s))$, if and only if conditions

$\Phi_{3.3}[1+ \cos\theta]0.02^2 + [\Phi_{3.2}(1+ \cos\theta) + \Phi_{2.2}(1 - \cos\theta)]\, 0.02 + [\Phi_{3.1}(1+ \cos\theta) + \Phi_{2.1}(1 - \cos\theta)]\, 0.0198 \le 2$

$\Phi_{4.4}[1+ \cos\theta]0.02^3 + [\Phi_{4.3}(1+ \cos\theta) + \Phi_{3.3}(1 - \cos\theta)]\, 0.02^2 + [\Phi_{4.2}(1+ \cos\theta) + \Phi_{3.2}(2 - \Omega(s) - \cos\theta)]\, 0.02 + [\Phi_{4.1}(1+ \cos\theta) + \Phi_{3.1}(1 - \cos\theta)]\, 0.0198 \le 2$

And

$\Phi_{5.5}[1+ \cos\theta]0.02^4 + [\Phi_{5.4}(1+ \cos\theta) + \Phi_{4.4}(1 - \cos\theta)]\, 0.02^3 + [\Phi_{5.3}(1+ \cos\theta + \Phi_{4.3}(1 - \cos\theta)]\, 0.02^2 + [\Phi_{5.2}(\Omega(s) + \cos\theta) + \Phi_{4.2}(2 - \Omega(s - \cos\theta)]\, 0.02 + [\Phi_{5.1}(\Omega(s) + \cos\theta) + \Phi_{4.1}(2 - \Omega(s) - \cos\theta)]\, 0.0198 \le 2$

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